An Introduction to the Perplex Number System

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Abstract

The perplex number system is a generalization of the logical relationships among electrical particles. The inferential logic of the new number system is homologous to the inferential logic of the progression of the atomic numbers. An electrical progression is defined categorically as a sequence of objects with teridentities. Each identity infers corresponding values of an integer, units and a correspondence relation between each unit and its integer. Thus, in this logical system, each perplex numeral contains an exact internal representational structure; it carries an internal message. This structure is a labeled bipartite graph that is homologous to the internal electrical structure of a chemical atom. The formal logical operations are conjunctions and disjunctions. Combinations of conjunctions and disjunctions compose the spatiality of objects. Conjunctions may include the middle term of pairs of propositions with a common term, thereby creating new information. The perplex numerals are used as a universal source of diagrams.

The perplex number system, as an abstract generalization of concrete objects and processes, constitutes a new exact notation for chemistry without invoking alchemical symbols. Practical applications of the number system to concrete objects (chemical elements, simple ions and molecules, and the perplex isomers, ethanol and dimethyl ether) are given. In conjunction with the real number system, the relationships between the perplex number system and scientific theories of concrete systems (thermodynamics, intra-molecular dynamics, molecular biology and individual medicine) are described.

Key Words:

Chemical mathematics, perplex logic, synductive logic, meso-syllogisms, labeled bipartite graphs

I. Introduction

The empirical logic of the chemical sciences is widely used in molecular biology and clinical medicine but the formal logic of chemical calculations of biochemical structures such as DNA and of valence relations are obscure. Since the early twentieth century, chemical theory is rooted in the exact correspondence between the integers and the list of chemical elements, suggesting a formal logic is possible. In view of the breadth and depth of current computational methods, the absence of a formal chemical logic is surprising. In chemical practice, predicate logic is used to associate to each chemical element with a list of unique properties that distinguish every element from all other chemical elements [**1**]. Most such properties are expressed in terms of the real number system. However, the nature of relationships between atomic numbers as the source of chemical information (structures and valence) and the real number system is unclear.

The conundrums include the formal numerical basis for the calculations of valence and of diagrammatic structures from atomic numbers. It is well known that the usual arithmetic operations of the real number system cannot be applied to the atomic numbers because this would violate the fundamental principle of chemistry that every element preserves it chemical identity during the processes of chemical change. An exact discrete mathematical logic for chemistry would open the opportunity for the powerful methods of mathematics to be applied to both molecular biology and clinical medicine. The absence of rigorous mathematical methods for describing biochemical processes (such as metabolism, physiology, mutation, reproduction, message encoding and decoding) severely impedes the development of exact theories of biological and biomedical processes.

The motivation for this work comes from three distinct sources, one mathematical, one biological and one chemical. Mathematicians have sought to address biodynamical issues from the perspective of continuous functions within a categorical framework. However, Robert Rosen, in ***Life Itself***, argues that life is beyond mechanical categorization [**2**]. In contrast to the categorical analysis of Rosen, Ehresmann and Vanbremeersch, in ***Memory Evolutive Systems***, demonstrate the possibility of using category theory for describing conscious decision-making [**3**]. The present work focuses on discrete mathematics, adapting facets of the diagrammatic logic of categories that exhibit exact calculations. Biochemical motivations originate in the fact that a single molecule of a mutagen may initiate biological processes that create a single substitution of one DNA base within a single cell, Chandler, 1973, [**4**]. This phenomenon is a deep conundrum from the perspective of the putative universality of chemical thermodynamics and the law of mass action. Pharmacological and toxicological studies indicate that trivial doses of a chemical agent can induce long-term changes in the health status of individuals, such as carcinogenesis. (Chandler, 1983, 1987) [**5**] [**6**]. Analysis of these three applications of mathematics to living systems all led to a common intractable dilemma, the mathematical nature of the logical operations on atomic numbers and chemical structures.

This paper presents an introduction to a discrete number system that expresses the mathematical logic intrinsic to chemical elements. It focuses on two logical operations of electric attributes to construct calculations. The diagrammatic logic developed for representing both ionic and covalent chemical structures is expressed formally in the categorical terms of graph theory and a special inferential method for calculations. The diagrammatic logic may be inclusive of the middle terms of pair's premises as a method of creating new logical diagrams.

The starting point for the mathematics of chemistry is the electrical characteristics of each atomic number [**1**]. It is these electrical attributes that create the basis for the correspondence between the atomic numbers and the perplex numbers. Two logically distinct types of electrical particles (electrons and nuclei) exist in every atom. (The masses of isotopes are of secondary consideration.) Consequently, the necessary logical structure for the chemical sciences is a binary logical structure such that the computations preserve the identity of the electrical attributes of every particle while concomitantly providing a basis for numerical calculations of relations. In other words, chemical computations constitute relational merological algebra.

A principle objective of this paper is to generalize from empirical scientific logic to an abstract natural binary number system. In order to avoid conflation with the complex number system (another binary number system albeit based on the real numbers) this new number system is termed the perplex number system. The nomination of the system as a natural system refers directly the correspondences with enumerations of electrical particles. Similarly, the nomination as a binary system emphasizes the separation of the components into classes of abstract objects, homogeneous parts and heterogeneous parts, units and integers, respectively. The homogeneous parts (units) are symbolically indistinguishable. The heterogeneous parts (integers) are both symbolically distinguishable from one another and have unique properties. Neither quantity can be substituted for the other form of quantity.

This number system is separated into two distinctive types of objects, the **perplex numerals**[1] and perplex numbers. The perplex numerals serve as precursors for all perplex numbers. Any mutation, any composition, any operation, any change of a perplex numeral by a logical operation creates a new object, a perplex number. Formal logical operations use the perplex numerals as the primitive source of counting of relations. The logical antecedence of the ordered list of perplex numerals serves as a source of heritable properties for operations and compositions. Perplex mutations, as an expression of change, are analogous to biological mutations as heritable changes of properties or capacities. The immediate motivation for this nomenclature is to create an exact logic that is homologous to the diagrammatic logic used for chemical structures. The relations between the perplex number system and the nomination of such structures of matter are expressed as labeled bipartite graphs.

This paper consists of six sections and a glossary. This introduction provided contexts – logical, mathematical and chemical – for placing this number system in perspective. The next section constructs the logic of the electrical progression as the basis of the perplex numerals. The first two mutative operations of the perplex numerals, transpositions and circules are introduced in sections three and four respectively. Section five introduces the synductive logic of one sort of perplex conjunction. The closing section addresses the origins of the empirical logic of the perplex number system and its relations to other disciplines. The inferential logic of the perplex number system must use terms that are closely associated with similar, but logically distinct concepts of the real number system. A glossary of terms for the perplexed number system follows the last section.

II. **The Perplex Numerals.**

The aim of this section is to compose, from simple symbols, an electrical progression that contains all of the perplex numerals. This electrical progression is, metaphorically speaking, a generalization of the abstract logic of generating chemical atoms from electrical parts. All perplex numbers will emerge from these perplex numerals by applying logical operations.

Before undertaking the construction, the issue of the terminology must be addressed. In practical calculations, the perplex number system and the real number systems are used concomitantly in scientific discourse. Yet, a distinction between the logics of methods of calculation and the modes of communication is necessary. Exposition of the perplex number system necessitates a terminology that separates the discourse for perplex number, inference and operations from the discourse of number, logic and operations of other number systems. Distinguishing the calculations on electrical particles from calculations on pure abstractions of time and space is necessary. The usage of the same symbols for integers to count the size or number of entities or objects in both systems appears unavoidable. The same argument holds true for the usage of number symbols as substrates for describing logical operations.

Nevertheless, I seek to avoid polysemy (the coexistence of many possible meanings for a word or phrase), to the largest extent feasible. To this end, two series of terms from the same Latin root, **ferre**, (to bear) will be used for the two logical systems. The terms of relate, relation and reference will be used for real numbers and in a general mathematical context. The parallel terms illate, **illation**, and inference will be used specifically for the perplex number system. The terms collate and **collation** will be used for two or more perplex numerals combined to form a totality or abstract whole with closure. Metaphorically, a collation is vaguely similar to the notion of a collection of functions on a collection of variables in the real number system. Further, the terms mutate and **mutation** (from L. mutare, change) will be used exclusively for describing a change of illations in the perplex number system. This is analogous to the biological usage of the term “mutate” where one or more properties of the subject are changed while other attributes of the message remain unchanged.

At the end of this section, some of the relations and illations between the properties of the perplex numerals and the parallel regularities and irregularities of the valence of atomic numbers are tabulated for the reader. It is this practical distinction in the regularities and irregularities of operations that must be addressed to develop a consistent formal logic for operations on the atomic numbers.

Construction of the perplex numerals

The start of the construction is with a mark as a symbol [see Mac Lane][**7**]. Sequential iteration or repetition of marks generates lists of marks (or messages).

1.
1,1.
1,1,1.
…

The writing of another mark indicates a new position in the list. The marks can be associated together to form a single entity. The count or size of each unique list of marks is named as a number or, more formally, as an integer, a thing complete in itself.

1.
2.
3.
…

This second list marks an electrical particle of a different type. The perplex number system consists of illations or inferences between members of these two progressive lists. The indistinguishable marks of the first list are termed units. The distinguishable marks of the second list are irreducible symbols. The two lists correspond or are comparable by size or count in the sense that one unit infers the integer one; two units infer the integer two, three units infer the integer three, and so forth. But each list communicates a different message; that is, members of two lists cannot be substituted for one another.

The first illation is the **parity** of the two lists. Using the standard logical notation for an inference, we write the following sequence of illations:

1.   1,1.   1,1,1.
--,   ----,   ------, …
1.   2.   3.

Because of the size of the terms for the integers and units are of the same count, we can also write the inverse inference:

1.   2.   3.
--,   ----,   ------, …
1.   1,1.   1,1,1.

Each member of the progression is composed from the consecutive logical terms of the two antecedent lists. Each member of the progression is termed a perplex numeral. Each numeral has the property of parity, that is, the size of the count of the integer is the same as the size of the count of units. This defines parity or electrical neutrality. (The concept of parity will play a critical computational role in all logical operations on perplex numerals and perplex numbers.) The iteration of the process of associating units, integers and illations is termed an electrical progression. Each term of the electrical progression is specified by this unique internal structure that confers an identity on every perplex numeral. (The logic of the term identity in this context is different from the arithmetic concept of identity.)

A mathematical graph (diagram) can be drawn for a perplex numeral of the electrical progression. Clearly, such a diagram must have an exact syntax that can be mutated by logical operations. Each diagram is composed from the units, integers and the illations that associate the two classes of concrete abstractions. Such a diagram is not merely a bringing together of logical terms to create an association, it is a collation of terms into a new object with a property of selfhood or **ipseity** (L. ipseity, self, as in I, myself). The collation entails the property of the connectedness of the logical diagrams. The diagrams for the first three numerals of perplex numerals are given in figure one.

Figure One.
The First Three Perplex Numerals
The three symbols represent the logical terms of integers, units (0) and illations (|). Each numeral is synonymous with a labeled bipartite graph, a logical diagram of the attachments of units to integers.

```
1          2          3
|         / \       / | \
0        0   0      0 0 0
```

Attributes of the teridentities

The identity of a perplex numeral consists of three structural components, the units, the integer and the illations associating integers with the individual units. Consequently, the term, **teridentity**, is used to associate the properties of a perplex numeral. The regularity of the growth of the individual logical terms corresponds exactly with the regularity of the growth of the counting numbers. A commutative association exists among these three properties for every member of the electrical progression, for each teridentity. Consequently, referring to each term of the electrical progression as a perplex numeral is consistent with definitions of ordering relations. The order property is a property essential for number systems.

The attributes of the logical order of the electrical progression differ from the simpler property of extension of the counting numbers. This construction is necessary in order to give the number system the capacity to build network from perplex numerals. The internal structure of the perplex integer includes the ordering of the numbers of logical inferences. This is necessary in order to correspond with the intrinsic properties that collate the two types of electrical particles. From these facts, one implies that the associative and distributive laws of the real number system are not valid for the perplex number system.

The construction of the electrical progression started with a mark for an electrical particle of one type. Consequently, the absence of a mark does not signify an electrical particle and hence no “zero” exists within the perplex number system. The construction of the perplex numeral and its teridentity did not invoke any concepts beyond the symbols needed to enumerate electrical particles (marks, units, integers and illations.) Such concepts as time and space are secondary properties and are beyond the scope of this introduction. Other concepts, such as of mass, number of protons or neutrons within a particular nucleus, isotopes, and radioisotopes are also separate and distinct from the concept of the electrical progression of the perplex numerals.

The teridentity of a perplex numeral serves as a term for logical **conjunctions** and **disjunctions**, associative and disassociative operations. Chemical change is expressed in terms of mutations of teridentities. In contrast to the internal regularity of the electrical progression, the practical operations on the teridentities are highly irregular. The term “valence” (value) is used to express one aspect of such practical chemical operations. Examples of the irregularity of chemical valences of the first ten elements are shown in table one. One of the objectives of this paper is to use iterations of logical conjunctions and disjunctions to generate diagrams that express the empirical concepts of valence.

Table 1
Regularities and Irregularities of Chemical Calculations
The table lists the first ten elements and the numbers of the electrical particles, units, the integer, the illations and the categorization of relations based on the table of elements. One of the objectives of this

paper is to use exact perplex logical operations on perplex numerals to describe the origins of the empirical concept of valence.

Name	Perplex Numeral	Integer	Units	Common Valences
Hydrogen	1	1	1	1
Helium	2	2	2	0
Lithium	3	3	3	1
Beryllium	4	4	4	2
Boron	5	5	5	3, 5
Carbon	6	6	6	2, 4
Nitrogen	7	7	7	1, 3,5
Oxygen	8	8	8	1, 2
Fluorine	9	9	9	1
Neon	10	10	10	0

III. Relational Algebra of Illations

The constructions of the electrical progression created an ordered list of perplex numerals - paritious objects with balanced internal structures. In this section we describe a logical operation on pairs of numerals that preserves parity.

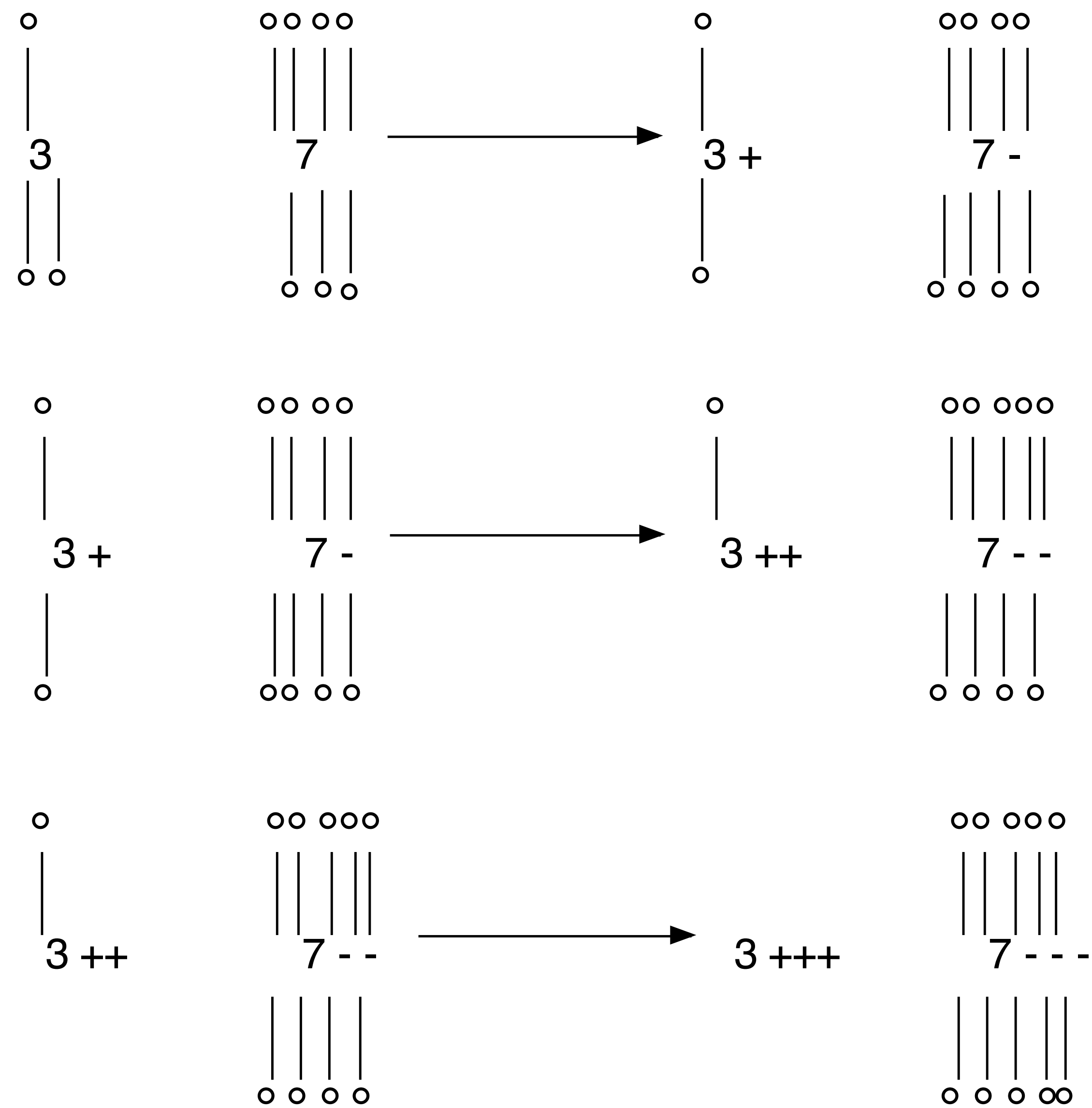
A logical operation of the perplex number system is a changing of pairings, a change of illations. Such changes are visualized directly among logical diagrams as change or mutation in the patterns of adjacencies. By a change in pairings is meant that any integer may increase or decrease the number of adjacencies with units. Symmetrically, any unit may increase or decrease the number of adjacencies with integers. The increases and / or decreases are concomitant logical operations such that parity is preserved over the whole of all participating entities. At least two entities are necessary for a logical operation; the logical changes of the entities are signified concomitantly. The changes of the pattern of inferences are termed conjunctions and disjunctions; the entities are mutated as a consequence of the logical conjunctions and disjunctions. The usage of the logical terms conjunction and disjunction refers to the illations of the perplex diagrams, the patterns of adjacencies. The aim of this section is to illustrate the calculation for one integer – unit pair. The next section illustrates the calculation for two integer - unit pairs. The following section specifies the perplex logic of parity preservation for these two classes of operations. Calculations for higher order changes (multiple concomitant mutations) are beyond the scope of this introduction.

A **transposition** operation is the simplest possible electrical operation of this number system. One unit is transposed from one integer to another. A disjunction disassociates one unit from its integer. A conjunction creates a new association with another integer. Clearly, a transposition preserves parity. The total number of integers, units and illations are preserved by the transposition operation. Categorically, the class of transposition operations could be termed a de-gluing and a re-gluing, or a detachment and a re-attachment or a separation and a reunion of a unit.

The diagrammatic logic of three *hypothetical* consecutive transposition operations is illustrated in Figure 2. The donor numeral loses a unit, mutating to a new internal structure with one less illation. The recipient numeral gains a unit and an illation, mutating to a new internal structure with one additional illation. The values of the integers are not mutated. The second and third transpositions of figure 2 follow the same pattern as the first with respect to the loss and gain of units, illations and structure. The size of a numeral limits the number of disjunctive operations. In the series shown, no further transpositions are possible, as the perplex number 3 without any attached units, cannot donate a unit. For inverse transpositions, the diagrams are read from right to left. For example, starting at the bottom right of Figure 2, three consecutive inverse transpositions would regenerate the original perplex numerals.

The donor capacity of a member of the electrical progression in the transposition operation is limited by its size. It cannot transpose what it fails to possess. In other words, each numeral has a unique capacity for transpositions operations, the integer value of the electrical progression, when the units of a numeral are exhausted, no further transpositions can occur. The example illustrates a critical feature of the internal structure of members of the electrical progression. As the concepts of inferences and quantification are intrinsic to the progression, the number of operations is limited by the size of the numerals. This is another critical attribute of electrical parity as an internal structural concept within the membership of the electrical progression.

A transposition that creates two different perplex numbers from two members of the electric progression, preserves parity. The traditional plus and minus signs associated with the integers are redundant information in the context of the diagrams.


3
7
3 +
7 -
3 +
7 -
3 ++
7 - -
3 ++
7 - -
3 +++
7 - - -

Section IV. The Circule Operation of the Perplex Numbers

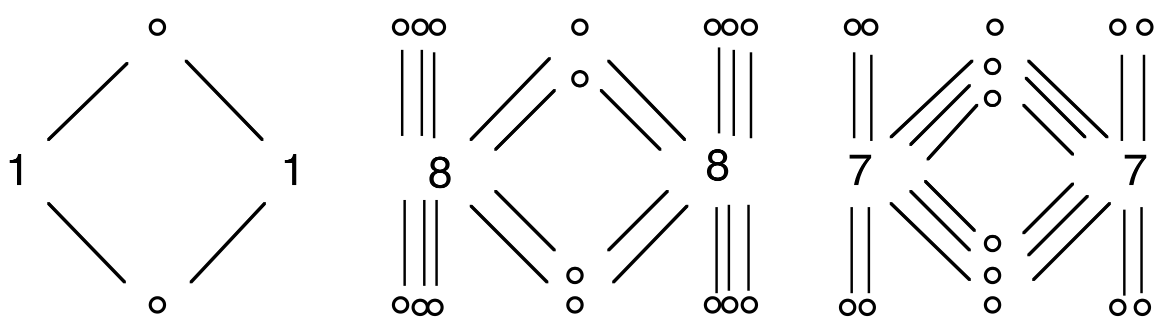
The **circule operation** is a conjunction of two perplex numerals to compose a circlet. It is a quaternary operation necessitating that two units create new relations with two integers, creating new electrical parity dependencies. Here, several examples of simple circule operations are given to illustrate some classes of logical diagrams formed. The circule conjunction, as a quaternary operation, has other special logical properties that are not merely an extension of two valid syllogisms into a sorites. The unique attributes of this logical operation are described in the next section of this paper.

As a consequent of the circule operation, several attributes emerge from the numerals into the circlet.

> The circlet constitutes a simple cyclic electrical network.
> As a circlet, no unique order of the entities can be specified.
> The conjunction of two independent perplex numerals creates a dependent pair.
> It conjoins two integer – unit illations to create a closed path with four illations.
> The diagram inherits all other properties from both parents, giving it a unique emergent identity.
> It is a closed path of illations between 4 entities as alternating units and integers; as such it is a labeled bipartite graph.
> The circlet constitutes an extension of the diagrammatic paths of both numerals.
> The number of new paths created by the circlet depends on the size of the perplex numerals.
> The inverse of a circule operation separates the circlet into antecedent integer – unit pairs.

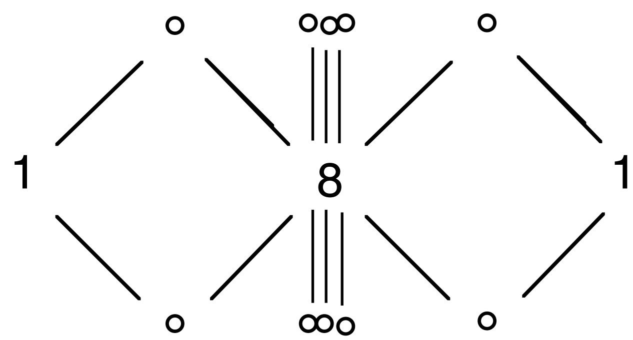
The novel qualities of the circule conjunction operating on different members of the electrical progression will now be illustrated with several examples. The first example, figure 3a, is the simplest possible example of a circlet formed on two members of the electrical progression. Figures 3 b and 3 c show the diagrammatic logic after two and three circule operations, respectively, on two other pairs of perplex numerals. (The nominations of the diagrams are hydrogen (1,1), oxygen (8,8) and nitrogen (7,7), respectively.) The iteration of the general logical operation is simple and direct, limited only by the number of units available from each numeral. As each example is composed of only one element, the “valence” of each is zero.

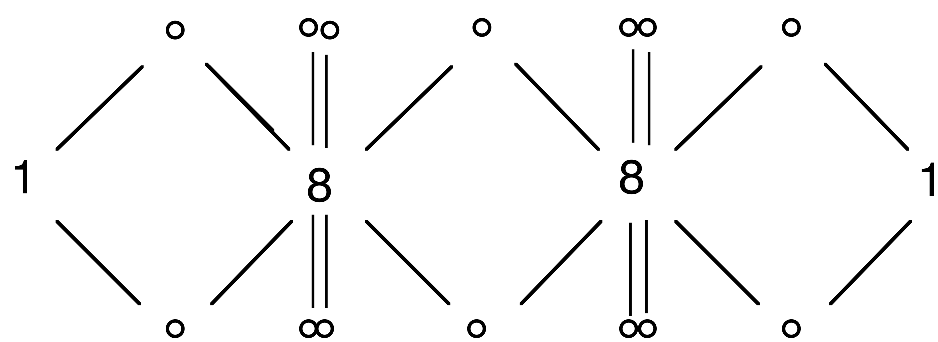
Figure 3.
The figure illustrates the consequences of the circule operation on three different pairs of perplex numerals. The circule operation is applied once in the first labeled bipartite graph, twice in the second and three times in the third. The resultant diagrams are known as hydrogen, oxygen and nitrogen in the language of chemistry.



In the second example, the abstract circule operation was applied repeatedly to two different teridentities, generating heterogeneous networks. Repeating the circule operation extends the set of illations already contained in the individual perplex numeral. The same two teridentities, one and eight, are used in the conjunctions to create two different perplex networks from the numerals (1,8,1) and (1,8,8,1). The concrete objects are water and hydrogen peroxide.

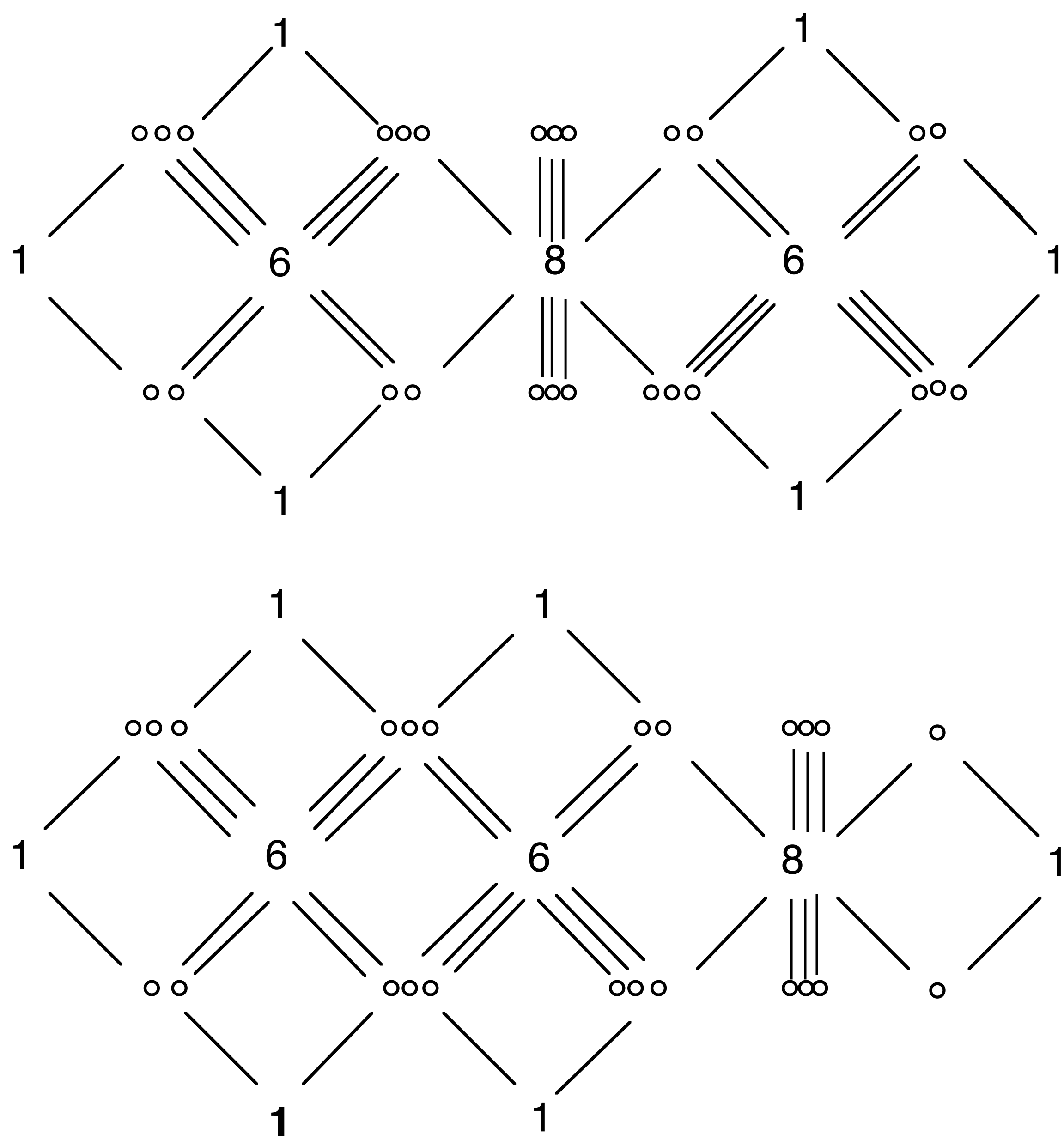
Figure 4 (a and b)
The diagrams illustrate the results of repeating the circule operation on the multisets of perplex numerals, (8,1,1) and (8,8,1,1). The chemical nominations of the labeled bipartite graphs are water and hydrogen peroxide.





In the third example of the circule operation was chosen to illustrate the consequences of the ordering of the choices of integers in the circule operation. The number of individual perplex numerals is nine, the number of integers is nine and the number of units is 26. Of all possible logical operations on a multiset of this size, two concrete examples are known. Each distinguished ordering of inferences on the same set of perplex numerals creates a different perplex network. (The chemical term, isomers, refers to the existence of two distinguished networks created from the same multiset of perplex numerals. Note that one isomer is symmetric while the other is not.)

Figure 5.
The two categorical diagrams that emerge from repetitions of circule operations on the same multiset (8,6,6,1,1,1,1,1,1) are drawn. The same number of circule operations (8) is used to construct the labeled bipartite graphs from the same multiset in both cases. The example illustrates the diagrams of two of possible isomers of this multiset. The chemical nominations of the diagrams are dimethyl ether and ethyl alcohol (ethanol). Note that one diagram is symmetric and the other asymmetric.



Attributes of the Circule Operation

As a general logical operation, the circule operation can be iterated over any collection of perplex numerals, any multiset. As the count of the multiset increases, the number of possible paritious diagrams increases. In view of the boundless combinatorial possibilities of perplex numerals and the boundless possible orders of operations, a systematic method is needed to index the theoretically possible entities arising from repetition of the circule operation.

Two sorts of indices are needed. The first sort is simple; the second appears to be intractable. The first index is simply a listing of the totality of all the numerals participating in the operations. This is a multiset. We will call this list the molecular word. A molecular word is characterized by two factors. First is the list of all the distinct unique individual perplex numerals present. The second factor is the number of repetitions of each numeral in the collation. Together, these two factors allow one to calculate the molecular parity value. The term multiset is used to describe the combination of the two interdependent measures of the size – the list of individual numeral and the count of each numeral in the list. The partition of the molecular word creates a multiset from the diagram.

The second index is needed to specify the possible ordering of circlet operations in relation to classes of multisets. As the number of numerals conjoined into a network diagram increases, the number of possible arrangements of the circlets increases irregularly, depending on the capacity of the numeral for circule operation and the number of multiple circule operations on pairs of numerals. The intrinsic irregularity of pairing of combinations of odd and even numerals of the electrical progression must be considered. These irregularities have resisted systematization.

Existential logic.

At this point, a lingering terminology issue must be addressed. It is necessary to separate the pure abstractions of perplex number theory that are merely mental images from objects that exist in the world with empirical tangible and analytically measurable properties. Without describing either how one comes to such practical decisions, we will simply introduce the term "ipseity" (L., ipseity, the thing itself) to be the individual known to exist concomitantly as both a pure abstractions of the mind and as a concrete object in the world outside of our minds. Since the generating functions of the perplex numbers generate mainly objects of abstraction, it is an empirical question to be answered by facts, not abstract conjectures. A subset of these logical abstractions, termed ipseities, corresponds with articles of commerce (hydrocarbons, carbohydrates, drugs, food additives, flavors, essential oils, natural products, vitamins, biochemicals and so forth.) The identity of an ipseity is shown by the measurement of unique properties such that it is distinguished from the attributes of all other multisets and all other ordering illations of the same multiset (isomers).

Of the labeled bipartite graphs given in the figures in this paper, several diagrams are ipseities and others are not. In figure one, only the natural ipseities of the perplex numerals are the teridentities 1 through 92. The existential perplex numerals have been extended to about 120 by artificial means. In figure two, the three diagrams of the symmetrical pairs of the numerals 1, 8, and 7 (hydrogen, oxygen and nitrogen) are well known ipseities. Of figure three, only the initial perplex numerals 3 and 7 exist; all others labeled bipartite graphs of figure three are non-existent. Of figure four, the two perplex numbers are nominated as water and hydrogen peroxide, both ipseities. The diagrams of figure five were identified as ipseities in the text.

Section V. Meso-Syllogisms as Logical Inferences in the Perplex Number System

The electrical progression and the logical operations of the perplex number system are abstract generalizations of empirical observations. A distinction between these logical operations (transpositions and circules) on the natural perplex numbers and the logic of arithmetic operations on artificial real numbers exists. One aspect of this syntactical distinction is asserted in this section. The roots of this critical distinction arise from the inferential grammar used to associate logical terms of propositional statements. Aristotelian **syllogisms** were the first systematic theory of inferences ([**8**]). Porphyry, using, the ten categories of Aristotle, introduced logical trees as a method of categorizing individuals, species, and genera. Since the circlets of conjunctive operations on perplex numerals are certainly not trees (figures 3, 4, 5), the question arises as to the quantitative logical structures of categorization of individuals, **species** and **genus** (or genera). The syntax of the circule operation creates new modes of inference from conjunctions that are different from modes of inference of syllogisms. A necessary property of the circule conjunction is the emergence of new order relationships such that an unlimited number of perplex numerals can be conjoined in space (see examples in figure four and five.) This conflicts with the necessity of syllogistic logic to eliminate one term. After briefly summarizing the nature of a syllogism, the logical forms of the circule and transposition operations will be developed from two term propositions.

A valid syllogism infers a conclusion from two premises [**9**]. (Boolean logic assigns syllogisms to arithmetic operations.) As a grammatical object, the propositional structure of a syllogism consists of three sentences – a major premise, a minor premise and the necessary conclusion. Each premise contains two terms, one term being common to both premises. The entailments of the logical concepts of all, none and some are related to one another by the classical square of opposition. For example, in the first square of opposition (termed Barbara), a valid entailment has the following symbolic form.

"All S is P."
"All P is M."
"Therefore all S is M."

The following semantic example illustrates the essential form of a syllogism.

All men are mammals.
All mammals are animals.
Therefore, all men are animals.

The conclusion omits the middle term, the term mammals. The verbs of the three propositions are predicates. The absence of the middle term from the conclusion removes the relations explicitly stated in the first and second propositions. In other words, the relation of the conclusion, S is M, is not necessarily dependent on the intervening term P. The common term of the syllogistic premises is disassociated from the conclusion. *If syllogistic logic were applied to premises with members of the electrical progression as terms, the principle of the conservation of matter and preservation of parity would be violated by the elimination of the middle term.*

One essential aspect of the perplex number system (as specified in the parity principle, which is a restatement of the conservation of matter principle) requires that the inferential operations preserve all components of perplex numerals of the electrical progression. In addition, the transposition operation and the circule operation, as asserted in sections 3 and 4, preserved all the terms (units and integers) of the perplex numerals. Consequently, these operations cannot be based on the deductive syllogism or on the classical form of predicate logic.

An abstract logical principle is embedded in the routine usage of chemical symbols. Formally, the abstract inference resembles a syllogism. It differs from a syllogism by preserving the middle term in the conclusion. This heretofore unstated logical principle is termed a **meso-syllogism** (L. meso, middle). Like a syllogism, a meso-syllogism is an inferential term logic, formed from two propositions, each containing two terms, one common to both premises, and a conclusion. The reasoning of a meso syllogism also reaches a conclusion from two propositions; however, the middle term is not eliminated. The verbs of the propositions in meso-syllogisms are not predicates in the traditional sense of predicate logic. Rather, the verb couples two terms together into a pair; the copula creates a new object that contains all three terms of the premises. Such a pairing may be thought of as an associative pairing, as a gluing, as an attachment, or as creating an adjacency relation between terms of the propositions.

Let A, B, and C be symbols of three independent terms. Then, an example of a meso-syllogism is stated in the following three sentences.

A is adjacent to B.
B is adjacent to C.
Therefore, B is adjacent to both A and C.

Symbolically, we can write the three sentences – the dash signifies adjacency - as
A – B.
B – C.
Therefore, A-B-C.

The semantic and symbolic examples give the logical form of the meso-syllogistic preservation of all three terms of the two premises. Grammatically, the verb is a copula. Alternatively, the meso - syllogistic conclusion can be stated as concrete relations implying ordered associations of three terms into an ordered entity.
Both A and C are attached to B.
B is glued to both A and C.
Both A and C are associated with B.

In all cases, the intent of the propositional statements is to use the copula (verb) to signify the coupling of components into a new abstract object, a new whole, which inherits the identity of the parts as components and specifies a sequence for the three terms contained in the conclusion.

The conclusion of a meso-syllogism forms an ordered relationship among the three terms given in the two premises of the meso-syllogism. This form of associative logic is termed a **synduction** by analogy of with deduction, induction and abduction, (Greek, syn -, with.) Symbolically, we can write the associative relations among the terms as A-B-C or as C-B-A. The order is specified without specifying an origin or a direction. The inference removes the possibility of concluding that the order of the three terms is B-A-C, B-C-A, A-C-B or B-C-A.

In the following examples, the exposition extends the associative logic of synduction to the transposition and circule operations of the perplex numerals of the electrical progression.

The application the synductive logic of a meso-syllogism to a perplex operation starts with the fact the internal structures of a perplex numerals and numbers exist as inferential pairs of parts. (This is explicitly stated in the diagrammatic logic.) Each term will be a pair that is glued together from two sub terms. Now, consider a meso syllogism formed from a pair of premises when both premises contain a pair of sub terms. That is, the starting terms are a pair of pairs (POP.)

Let the starting terms be A-B and C-D.
We form a proposition by coupling the terms to compose a meso-syllogism:

Sub term B of the term A-B couples to sub term C of the C-D term.
Sub term D of the term C-D couples to sub term A of the A-B term.
Therefore, the cyclic association is created, -A-B-D-C-, or,

A – B
 | |
C – D

Analogously, a meso-syllogism can be applied to a pair of perplex numerals.
For example, let
1(a) – O (a) and
1(b) – O (b)
be a pair of the first perplex numeral. (The indices a and b signify only that two distinct representations of the same perplex numeral are being considered.)

We write the two premises of the meso-syllogism as:
1(a) attaches to O (b).
1(b) attaches to O (a).
Therefore this synduction creates a circlet with alternating terms of units and integers.

O (a) – 1(a)

| |
1(b) – O (b).

Thus, the quaternary circule operation is a meso-syllogism on a pair of perplex numerals where one unit and the integer are from each member of two independent perplex numerals. Note that this meso-syllogism has the desired attribute of preserving the parity of the primitive pair of terms. A circlet, as defined in section 4, is the consequences of the synductive operation between two perplex numerals. In practical terms, a meso-syllogism on two perplex numerals defines the order of covalent bond formation.

The transposition operation is also dependent on statements that are expressed as copulated propositions. The propositions are stated in terms of the actions of detaching and attaching. Let the two premises be that with three terms: perplex numeral (A), perplex numeral (B) and one of the units glued to a perplex numeral A.

Numeral A detaches one unit.
Numeral B attaches one unit.
Conclusion:
Numeral A and Numeral B have changed by one unit.

The synductive operations of the perplex number system mutate or transform illations between and or among numerals. In this introduction, only first order (transpositions) and second order (circule) operations were introduced. Extension to higher orders is obvious. In addition, the empirical observations show the potential for very high order operations (catalysis).

The order of the synductive logical operations illustrate the deep structure of the perplex number system lies in the conceptualization of change (mutation) as a consequence of the interior mode of extension of the electrical progression.

VI. Conclusions and Discussion.

Order

Two symbolic abstractions of the electrical attributes of the atomic numbers were used to construct a number system. One symbolic attribute was abstracted as a mark; the other attribute was abstracted as an integer. Rotman considers marks and integers to be the “Ur-symbols” of mathematics. [**10**] The symbolic order of the electrical progression is generated from repetitions that associate the “Ur-symbols”, thereby creating the usual order property of increasing size of each successive numeral. The order property founds the properties of calculations.

Emergent Orders

The neo-Aristotelian abstractions of the perplex number system are founded on two sources of order. One source of order is encoded in the electrical progression. This type of order encodes transitivity. The second source of order is the order of logical operations on pairs of numerals of the electrical progression. This order encodes part-whole relations. It is a merologic order. The internal structural organization of the transitive order is a necessary precursor of emergence of the merologic order. The second type of order is encoded in neo-Aristotelian categories of individuals, species and genus. For any arbitrary multi-set of individual perplex numerals, a species and a genus exists. A species is generated from an order of circule operations on pairs of perplex numerals of the multiset. The genus consists of the family of species, the isomers, of the particular multi-set. In the example of figure 5, starting from a common multiset, one emergent ordering of operations produces a symmetric diagram and another emergent ordering produces an asymmetrical diagram – two species of the same genus, two isomers. In addition to pragmatics of the individual and the correspondence of illations, structural closure in the perplex number system requires a global coherence to explicate the number of isomers [**11**]

The two sources of order of the perplex number system necessitate the unique syntax of synductive logic. The propositional operations of the synductive logic are repeated until the entire multiset is connected into a spatial network with specific paths connecting all numerals. The resultant diagram will show all the illations created from logical operations. This is logically analogous to an Aristotelian **sorites**, so that the ordered sequence of circule operations is a **meso-sorites**. A meso-sorites corresponds to a path of formal and material causes within Aristotelian logic. *The Same and Not the Same*, [**12**] by the Nobelist Roald Hoffman, gives many concrete organic examples relating chemical identity, chemical symbols, and material causality.

No necessity for the conception of infinite order of perplex numerals exists. One cannot draw a diagram with a non-finite number of symbols and no particular advantage is gained by imagining that one can do so. The ordering of internal structural operations (transpositions, circules and other operations not defined here) is bounded by parity, ensuring operational and diagrammatic closure. Hence, the concept of infinity is not needed for this number system.

Comparison of Number Systems

The attribute of order creates the possibility for exact logical operations and calculations. Older number systems serve as a basis of evaluation of the novel characteristics of the perplex number system.

Historically, the Greek tradition created the precedence for the intimate correlations of concepts of number and geometry. For instance, the triangular numbers, the polygonal numbers and the pyramidal numbers were well-known mathematical objects to Greek mathematicians. For a discussion of Greek number systems, see Conway and Guy [**13**]. Such number systems did not support generalized logical operations on categories.

The real number system is co-founded in the regularity of the geometry of the line. One imagines that the infinite extension of the “real number line” contains all possible positions that exist or that be may calculated to exist, even if they can not be demonstrated. The principle identity symbols, zero and one, must be defined before the usual arithmetic laws can be applied. Further, the infinity symbol must be added to ensure that arithmetic operations in the real number systems achieve operational closures. Number and geometry are mutually encoded attributes in the complex number system.

The following table (Table 2) provides the reader a partial semantic guide for comparing some of attributes of perplex number system, as described in this paper, and the real number system.

Table 2		
Type of Progression	Electrical	Arithmetic
Name of Number System	Perplex	Arithmetic
Origin of Number System	Mutual encoding of number, electricity and illations	Mutual encoding of number, geometry and relations
Propositional Grammar	Statements conjoined by copula	Statements conjoined by predicates
Propositional Logic	Conjunction, disjunction and meso - syllogism	Conjunctions, disjunctions and syllogisms
Attribute of Identity	Objective identity as distinguishable from one another	Identity as a logical operator
Semantic Closure	Finite	
Syntactical Closure	Parity, merologic (Parts / Whole)	
Operational Closure	Memory of illations	

Scientific Consistency

The electric attributes of individual elements are the only attributes that grow consistently from element to element such that a regular order is consistent with an abstract number system [**1**]. Other attributes, expressed in the real number system, such as mass or atomic size, are irregular such that logical operations will not correspond with unit changes of "valence". Consequently, a formal symbolic logic for the chemical sciences is possible only by inchoative principle of **electricity**. Electricity is an undefined initiating concept in this context. By way of comparison, set theory requires two undefined initial concepts, member and class, to enter into both term and predicate logic.

Eulerian graph theory is widely used in mathematical chemistry. [**14**] [**15, 16**] [**17**] The Eulerian graphs are not defined in terms of an electrical progression. The perplex number diagrams are readily transformed into Eulerian graphs by simplifying the associative rules for association of symbols.

Consistency in communication of scientific quantities

The perplex number system and the real number system are complementary methods of expressing numeric quantities. Both are abstract systems for expressing methods of reasoning and calculations. The complementary roles of illations (as internal structures of molecules) and relations are established by current scientific practice. Neither the natural nor the artificial number systems are a material theory of reality.

The perplex logic for illations opens opportunity for improved communications between the chemical sciences and other disciplines.

Two formal measures of quantity of matter exist in current practice. International agreements and treaties establish the two systems of measure. These two notational systems are both integral to the practice of science. One system of measure is governed by the Systeme International d'Unites (SI) (see: http://www.bipm.fr/en/si/. It was designed as a legal code to encourage honesty in commerce. The other system, over-seen by the International Union of Pure and Applied Chemistry (IUPAC) is a formalization of chemical practice. (see: http://www.chem.qmul.ac.uk/iupac/)

The two systems offer two independent logical approaches with respect to the quantification of matter. One is based on a generic concept of matter as mass, the other on the concept of matter as merological wholes systematically composed from parts into unique species.

The IUPAC system, based on the atomic numbers, is a system for distinguishing each unique form of matter with respect to the relationships between the parts and the whole and assigning it a unique name that distinguishes it from all other quantities of matter. No algebraic requirements can be imposed on the individual units of matter or substance since the chemical symbols are used to designate the individual units of composition. The perplex number system expresses the quantitative illations that generate these identities from the atomic numbers.

The SI presupposes that the fundamental generic quantities of commerce can be algebraically independent of one another, each being treated as variables in systems of equations.
"The base units are a choice of seven well-defined units which by convention are regarded as dimensionally independent: the metre, the kilogram, the second, the ampere, the kelvin, the mole, and the candela. Derived units are those formed by combining base units according to the algebraic relations linking the corresponding quantities."
In other words, the SI system focuses on the predicated properties of matter, each value of a property being a measure within the real number system; the atomic numbers are excluded. For example, the property of mass is a generic quality of matter, represented as a singular geometric point, independent of the number of illations that organize its structure.

Role of Illations and Relations in Theories of Dynamical Systems

The two systems of quantification of matter may generate logical conundrums in the interpretation of scientific data that require consideration of both substance (expressed in the IUPAC / perplex number system) and the properties of substances (expressed in the SI / real number system.) I list three examples:

1. Chemical thermodynamics
Thermodynamics originates in the study of heat and the ideal gas laws. The seven variables of thermodynamic systems (volume, pressure, temperature, heat, entropy, Gibbs energy, free energy) are related in a bilinear group of equations describing the properties of macroscopic system equilibrium. Chemical symbols are not admitted into these equations of thermodynamics. Neither are the discrete symbols of perplex numerals or perplex numbers. In other words, the second source of chemical order, the emergent order of chemical species, is excluded from the formal (predicate) logic of thermodynamics.

However, an indirect connection between chemical symbols and thermodynamics exists. The indirect connection is via a relationship between the equilibrium constant for a chemical reaction and the free energy by the classical equation, ($\Delta F = - RT \ln K(eq)$). The equilibrium constant is ratio of concentrations (numbers per unit volume) of molecules at equilibrium. It can be valid only for very large populations of molecules. From the perspective of the perplex number system, the calculation of the equilibrium constant requires switching contexts from the perplex number system to the real number system.

The molecular biological theory of genetics is based on functional illations between sequences of DNA, RNA and proteins as message bearers within an elaborate system of communication. In the absence of formal methods for entering chemical symbols into thermodynamic functions, a derivation of correspondence relations between a mutation of a DNA sequence and mutated organisms is an intractable dilemma. The conundrum of the suitable thermodynamic treatment of reactions of individual DNA bases exists as a consequence of the logical separation of usage of symbols to represent matter from the symbols to represent the properties of matter. This separation of system of quantification starts to address the basic question of the nature of biological mutations [**4**]. Biologically based information theoretic approaches to DNA structure have also provided useful analytic insights into the individuality questions. (See: http://www-lmmb.ncifcrf.gov/~toms/schneider.html)

2. Intra-molecular Dynamics
Dynamics, the study of time or temporal processes, can be imposed on the chemical illations. Two forms of temporal relations are common, internal temporal relations among electrical particles and external chemical relations that mutate structures. Only intra - molecular properties are addressed here. Software packages, such as Amber, and CHARMM, are routinely used to approximate the spatial – temporal behavior of electrical particles or collations of particles within a given structure. Generally speaking, the aim of these algorithms is to seek local energy minima in terms of possible geometries. The equations of motion presuppose the real number system and approximations of functions. Exact mathematics is often intractable. However, given a logical diagram as the initial conditions, highly useful approximations can be calculated.

3. Molecular biology and medicine
Chemical analysis of living systems was vigorously pursued during the 20 th Century. The attributes of several million unique chemical structures have been identified and collated with biochemical information flows. Consequently, the scientific sub-disciplines of medicine (anatomy, genetics bacteriology, nutrition, toxicology, pharmacology) are now all viewed through the common lens of molecular structures at various spatial and temporal scales. The consistency among the sub-disciplines of medicine emerges, in part, from the genetic code. As chemical code, the genetic code is a second order symbolization derived from the IUPAC system of quantification of matter. Further, it is well known that most physiological dynamics are closely correlated with terms of dynamics of chemical variables. The recent emergence of genomic and personalized medicine is grounded in the difference of identities of chemical structures (isomers) from individual to individual. The concrete abstractions of the perplex number system provide a formal exact logical method for the description of mutations of organisms from mutations of diagrams.

In each of the three cases, thermodynamics, intra-molecular dynamics and biodynamics, the chemical system must be defined in terms of the chemical structures before models of continuous functions (time, space, volume, forces, geometries, and so forth) can be applied. From the perspective of scientific consistency, the perplex numerals are the ground for the emergence of artificial number systems.

Summary

The perplex number system was constructed as a mathematical abstraction of chemical reasoning. The objective is to formalize the numerical calculations within and between species of matter, such that the chemical information can be re-coded into biological information. Specific logical operations are used to specify electrical relations rather than the generic relations of mass and valence. The four interrelated components of the perplex number system described in this introduction are: 1. An electrical progression as a universal reference source of irreducible numerals with internal structures, 2. Conjunctive and disjunctive logical operations, 3. An emergent propositional term logic (meso-syllogistic logic), and 4. Logical diagrams for visualizing structures and conjunctive and disjunction operations. Modules under construction include the introduction of time and the introduction of optative grammar to describe modes of choice such as illations of catalysis, the emergence of life, and biological reproductive systems.

Acknowledgments: The author expresses his gratitude to two anonymous reviewers for very helpful comments and suggestions that materially improved the exposition. Stimulating discussions with Bert D. Chandler are gratefully acknowledged.

Glossary of Terms

1. circule operation: a quaternary logical operation that creates a cyclic diagram (a circlet) from two perplex numerals

2. collation: a connected multiset of perplex numerals

3. conjunction: creating a new logical diagram by conjoining two diagrams

4. disjunction: separating a logical diagram into two diagrams

5. electricity: an undefined term, a discrete attribute of chemical particles

6. ferre: L., to bear, to carry; base of verbs indicative of change / motion; such as ablate, collate, illate, prolate, relate, translate

7. genus: a collection of species

8. illation: a relation between a perplex unit and a perplex integer

9. ipseity: a species of matter that corresponds with both abstract attributes and concrete properties of a labeled bipartite graph and a diagram

10. meso-sorites: a sequence of meso-syllogisms connected by common terms in propositions; consequently, the sequence of propositions that generate the emergence of new symbolic networks by concatenating perplex numerals via meso-syllogisms

11. meso-syllogism: a term logic that creates new order relations from the three terms in two premises

12. mutation: a change of a diagram or of an ipseity

13. parity: the distribution of electrical neutrality of an perplex number among units and integers

14. perplex numeral: a member of the electrical progression

15. sorites: the concatenation of syllogisms with common terms

16. species: a member of a genus

17. synduction: the logic of emergence of new terms from simpler terms

18. syllogism: A reductive term logic described by Aristotle. Schoolmen developed the square of opposition as a method of categorizing the relations by predicate logic

19. teridentity: the concomitancy of three values (the units, the integer and the illations) of a member of the electrical progression

20. transpose: to move from one position to another

Addendum

After submitting this paper, the paper by I. Ugi, et al on "Models, concepts, theories, and formal languages in chemistry and their use as a basis for computer assistance in chemistry" [18] was called to my attention. The mathematical representations of chemical structures as matrices in the work of Ugi, et al are based on concepts of atoms and valence. Very roughly speaking, the symbolic representations of the perplex number system is readily reduced to traditional chemical representations by removing the requirements necessary to define the teridentity and representing each circlet as a chemical bond between two chemical symbols.

Even though the stated motivation is rather distant, an intuitive comparison is possible.

A principle common motive appears to be that we both start from the same intuitive notion that the encoding of a chemical structure is some sort of mathematical object. The goal is to give an exact description of chemical objects so that exact calculations can be made for chemical systems. This goal is elusive.

Ugi's principal motivation is intimately connected to computerizing the choice of optimal pathways for chemical synthesis. Deitz seeks to extend Ugi's work by intuitive judgments about "bonding systems." Both seek to compute chemical structures “in silico.”

In principle, both Ugi et al and Deitz papers seek to re-code traditional chemical structures by a new coding system(s) such that the resulting "trans-chemical" code can, in turn, be re-coded into binary code for electronic computation. Pragmatically, Ugi’s re-coding scheme can be used to select substrates for organic synthesis.

This work seeks to create a code such that chemical and biochemical relations and structures can be encoded. Thus, the necessity for the synductive logic of the particular is intrinsic to the codes of living systems. In so far as the contents of this paper are concerned, my work stops where the works of Ugi et al and Deitz begins. Consequently, for anyone skilled in these sciences, one merely translates from the “perplex number” code into traditional chemical code and hence into the codes of Ugi and Deitz and hence into digital codes. These are semiotic re-coding processes that change the information content of alphanumeric symbols.

The use, by various authors, of various synthetic symbol systems (encoding and decoding relations intrinsic to chemical and mathematical communications) with similar but not identical meanings for symbols is very confusing and leads to contradictions. We can do little to change the historical usage of coding symbols used in adjacent disciplines such as mathematics and chemistry. (Logical aspects of comparing several different synthetic symbol systems are addressed in a recent communication (Chandler, 2007)[19]).

To conclude this addendum, I repeat that this paper is merely an introduction to a wider theory of perplex numbers. A great deal of work remains to be done (for example, to include additional logical relations, to include additional diagrams, and to encode the relations of biochemical catalysis) before the perplex number system can serve as a mathematical code for living systems. The "bigger picture" will become clear in time.

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Endnotes

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[1] Special terms indicated by the **bold face** type are defined in the glossary.