Dynamical Concepts used in Creativity and Chaos[i]

Frederick David Abraham, Stanley Krippner, & Ruth Richards

Abstract

Basic dynamical concepts relevant to human creativity include those of stability, instability, bifurcations, and self-organization. Here we present the view that most creative bifurcations are from chaotic to chaotic attractors, and that such bifurcation are macro-bifurcations comprised of a cascade of micro-bifurcations whether in continuous dynamics or network-style models. Examples are drawn from the Ueda Explosion for continuous systems and the work by Langton; Packard; Mitchell *et al*., and Kauffman with NK Boolean networks. These typically are thus smeared in time and state-space. We suggest that the misunderstood phrase "edge of chaos" could be used for such a liberalized concept.

Key words: creativity, stability, instability, bifurcations, self-organization, "edge of chaos"

From the Leibniz-Abraham Correspondence

> *And for motion and temporal change in complex interactions, my "conatus" and "trace", which you call "vector" and "trajectory", have developed into attractors and phase portraits, thus extending my geometry of motion.*[ii]
> *If these are correctly and ingeniously established, this universal writing will be as easy as it is common, and will be capable of being read without any dictionary; at the same time, a fundamental knowledge of all things will be obtained. The whole of such a writing will be made of geometrical figures, as it were, and of a kind of pictures—just as the ancient Egyptians did, and the Chinese do today. Leibniz, 1666, De Arte Combinatoria*

Part I. Stability, Instability, and Bifurcations

Supposing you, a friend, or a patient has some behavioral feature or personality characteristic which you wish to change. Changing such an undesirable attractor we might call a dynamical bifurcation. For us, the most important part of the theory of dynamical systems is that of bifurcation, which means a significant change or transformation. The most common types of attractors are ***static***, ***cyclic***, or ***chaotic***. The most familiar types of bifurcations are ***catastrophic*** (an attractor appears or disappears); ***subtle*** (changing from one type to another); or ***plosive*** (exploding or imploding). We would call persistent behavioral patterns stable, whether maladaptive or healthy. To get from a less desirable trait to a more desirable one requires that one destabilize the system. We can see this by visualizing how dynamics treats such change. One can describe bifurcation properties by defining change mathematically or metaphorically, depending the extent to which you wish to formalize the system.

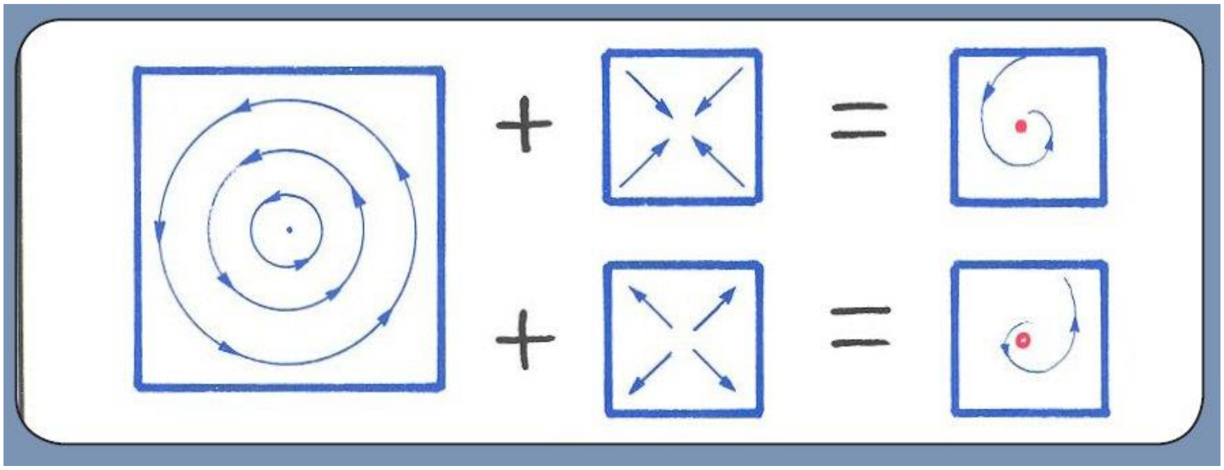


Figure 1. An unstable center can be perturbed into a point attractor or point repellor. Taken from Abraham & Shaw (2005, Fig. 12.1.8, p. 368.)

On the left is a dynamical (aka 'phase' and 'attractor-basin') portrait called a 'center'. The vectorfield of forces at each point in the state space cause the two variables to cycle over the same trajectory through the same starting point. The three cycles start at different points. Starting at a point nearby to those trajectories neither approach nor move away from that trajectory. There is no attractor or repellor in this portrait. This model has been used in ecology (prey-predator relationships, Lotka, 1924; Volterra, 1931) and family dynamics (a mother's tension and a son's aggression, Elkaïm *et al*. 1987). The vectorfield and its portrait shown here are extremely unstable. Why? Because the competing forces (say between the variables of the prey and predator populations, or the tension/aggression within the family) are rather equally balanced. Changing the constants amplifying and attenuating those variables, changes the vectorfield, as shown by the vectors in the middle pictures. The upper panel shows the added vectors pointing toward the center of the center, causing trajectories to spiral in to a fixed point attractor, a stable attractor, the portrait at the upper right. If the new vectors point away, a point repellor is created. At the bifurcation point for such ordinary bifurcations, there is no attractor. To get from one attractor to another requires changing one or more constants, with the system losing stability, and passing through the bifurcation point, if a new attractor appears. As the constant is changed further, it becomes more stable again. The phrase 'far from equilibrium' means just this, moving away from stability to instability and the potentiality for bifurcation.

Thus as claimed, destabilization is an agent of change. But chaos is not usually a feature present at the bifurcation for many classic systems. How can we justify suggesting that it can be? In most human systems, attractors are chaotic, preferably mid-dimensionally, and most bifurcations are between topologically different chaotic attractors. We here conjecture that with complex chaotic attractors, we can have some chaotic features remain, while others in the portrait do, as in the simpler cases, change at their bifurcation point. These could participate in the self-organizational capacity of the system to be creative.

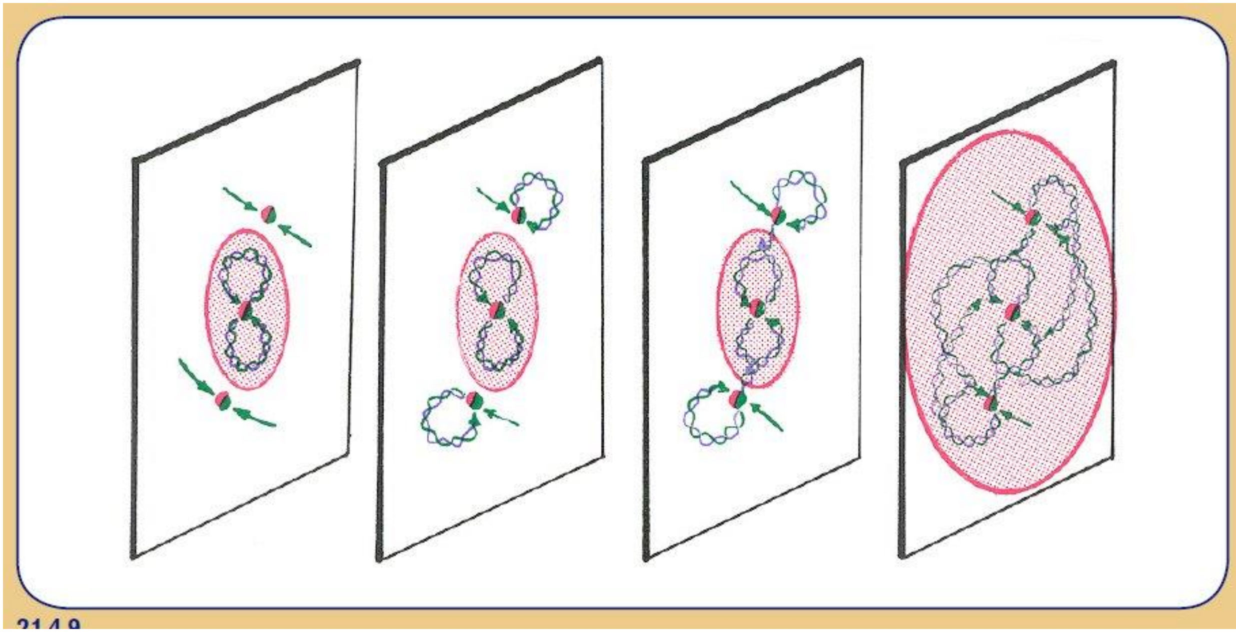


Figure 2. **Portraits of Ueda's Explosive Bifurcation, an example of a chaos-to-chaos bifurcation.** Taken from Abraham & Shaw (2005, Fig. 21.4.9, p. 609.)

"Ueda's Chaotic Explosion in 3D (1980). The attractor is in the red area. Outside of it are two saddles with homoclinic connections (from the saddle to itself). The chaotic attractor in the center is a tangle organized around a saddle. Trajectories from the outer saddles are attracted to the center saddle and, along with its own homoclinic connections, form the inset to the center saddle. Then the outset of the central saddle makes connections to the outer saddles, which are then part of the larger chaotic tangled attractor." (Abraham & Shaw, *2005*.).

You can see why it is called explosive, and see both the former homoclinic, as well as the new, heteroclinic tangles of trajectories. We contend that these chaotic elements contribute to, or perhaps better, characterize the creative process. The trajectories within the tangles sequentially bifurcate, being micro-biurcations that smear in time and space to constitute the macro bifurcation.[iii]

Ueda refers to this explosion as a "chaotically transitional process":

"Duffing's equation has not statistical parameters and every solution is uniquely determined by the initial condition. The appearance of statistical properties in the physical phenomena in spite of the perfectly deterministic nature of the equation is caused by the existence of noise in the real systems as well as in the global structure of the solutions. A bundle of solutions representing the chaotically transitional process appears in certain domains of the system parameters. The details of those stochastic regions have not been discussed as yet." (Ueda, 1980. P. 137 (425 in the Kyoto repository).

Whether one focuses on creative geniuses or everyday creativity, creativity depends on destabilizing the system. That statement we consider a necessary but not sufficient condition for creativity.

Part II. "The Edge of Chaos"

At first we didn't want to use the phrase 'edge of chaos' because we were convinced that few, including ourselves, knew what it meant, and that many used it in a very general sense without having any explicit notion of its definition. Even within the enclave at the Santa Fe Institute, Langdon, Farmer, Crutchfield, Packard, Hraber, and Kauffman *et al*. who developed the ideas, there were differences in usage. It deals with the patterns of activity in complex adaptive systems (CAS), which are two or more continuous or discrete dynamical systems that are locally interactive by means of their influence on each other's control parameters, more especially those called cellular automata (CA). These are discrete spatial systems (networks, lattices) with global properties much like a dynamical system. Most of the initial work was done with Boolean NK systems, wherein each of N nodes can be in one of two states, and be influenced by K neighbors. For these simple models, the system evolves synchronously in steps, updating the states of all nodes according to transitional logical rules. An example of a very simple system showing the relationship to dynamical system representations of variables (nodes), equations (logical rules), time-series representations, and trajectory representations in state space are shown in Figure 3. While such models have limited utility, they proved an important phase in developing network models and statistics. One of the first developments in statistics for Boolean networks investigated a parameter, λ, the proportion of the logical rules that yielded a given number of states (0's vs 1's as the usual coding for binary states). Langton (1990) found that as the proportion, λ, approached a critical value, $\lambda_c = 0.5$[iv], then the more complex the attractors became, going through a bifurcation sequence from point, to cyclic, to more random or chaotic as defined by a number of traditional information measures. Doyne Farmer first attached the label "edge of chaos" to this middle region near λ_c.

In actuality, there can be no such thing as true chaos as is made clear from the finite nature of all networks; all trajectories must be cyclic (Mitchell, 2009, p. 285), but some, cycles are so long as to be meaningless, but we treat the complex portions as chaos when they display certain properties, such as initial sensitivity to conditions, good behavior for the usual measures of fractal dimension and Lyapunov exponents when or if computed, and so on. "But if the state cycle is too vast, the system will behave in a manner that is essentially unpredictable." (Kauffman, 1995, location 1056).

Note the similarity of the discussion of the network transitions to Ueda's.

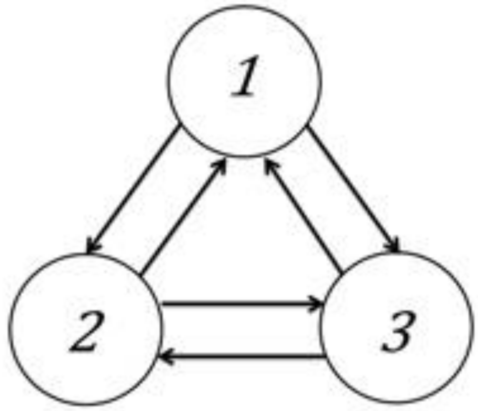
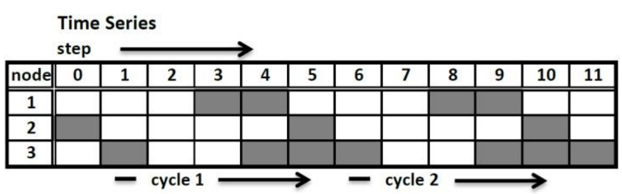
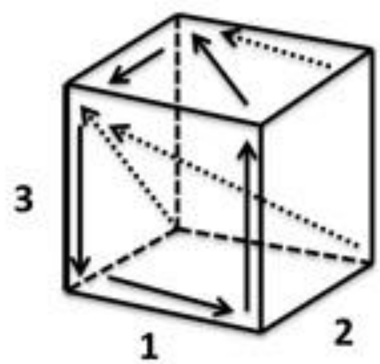
	node 1 = 1 if 2 ^ 3 = 0 node 2 = 1 if 1 ^ 3 = 1 node 3 = 1 if 1 v 2 = 1
System Diagram: Variables (Nodes) 1,2,3	The logical rules determining states at step n given the states at step n-1
	
Time series for the three nodes being synchronously stepped for 11 steps after arbitrarily choosing an initial state from among the eight possible at time step 0. It has a 5-cycle attractor, the first two cycles being indicated. The trajectory for this time series starts from the right rear lower vertex in the state space shown in the next panel. On is gray or 1; off is white or 0.	All possible trajectories in 3D state space. Right, back, and up represent the 'on' states for nodes 1, 2, 3. The solid arrows belong to the cyclic attractor; the dotted arrows are the single step approaches to the periodic attractor for the three initial conditions not on the attractive cycle.

Figure 3. Boolean 3,2 Network

Since logical rules have no parameters modifying variables, as with the ordinary differential equations of 'continuous ' dynamical systems, changing their effect must instead be accomplished by changing the logical rules themselves.

Packard (1998) investigated a statistical property of dispersion, the difference-pattern spreading rate γ (somewhat similar to Lyapunov exponents for continuous dynamics), as network trajectories evolved, also as a function of λ. As with Langton's measures, there was a region, though rather broad, at the mid region near λ_c which yielded higher λ's Mitchell, Crutchfield, and Hraber (1994) ran similar simulations and got a narrower range of elevated γ near λ_c. Because the equavalnce of details of these simulations, such as the genetic algorithms used to produce logical rules, procedures for selecting initial conditions for simulations, K, etc., Mitchell *et al*. caution that "more appropriate measures of dynamical behavior and computational capability must be formulated, and the notion of the 'edge of chaos' in CA must also be well defined." (p. 14). To complicate matters, she mentions that Kauffman (1995), who extensively pioneered the potential application of Boolean NK networks to genetic networks, also found these different types of attractors to be a function of N and K as control parameters, and also applied the edge-of-chaos terminology to the 'completely chaotic' or complex regime which he considered to be under self-organizational control. (Mitchell, 2009, p. 284-6).

But our complaint is not with the inability to provide such a good definition, but to the implication of such a phrase as the edge of chaos that the important bifurcations are from simple fixed point or periodic attractors to chaotic ones. We contend that most bifurcations are among differing chaotic regimes, and like the simulations of networks, or the simulations of dynamical explosive bifurcations like the Ueda, and that these bifurcations smear things in time, perhaps with local micro-bifurcations comprising macro-bifurcations. These could reflect the result of the interaction of local and global activities, suggestive of the sometimes controversial claims of Wolfram's Class IV behavior, a claim recently been made for neurobeedback (Bachers, 2012), and which could obviously characterize a host of mental and neural domains, to say nothing of other biological, ecological, physical, chemical, and social systems. Boolean networks were extensively investigated as perceptual models by Malloy. (Malloy et al., 2010; Malloy & Jensen, 2008).

Part III Self-Organization

Our use of the term self-organization is taken from that of Ralph Abraham and Shaw (1987) and can be characterized as a control parameter being a function of the state of the system. That is, for an intentional system, like the human, it means navigating in parameter space to change one's own behavior, thinking, mind, brain, etc. The system is usually under some control by both self and environment including external variables not formalized within the state or parameter spaces in a model of the system, as depicted in Figure 4. For a human, there is usually some struggle to achieve a creative chaotic bifurcation, even to get to the region near such a bifurcation, and would suggest that the term edge of chaos for the individual could represent the struggle to get there. Hopefully one succeeds in accomplishing the bifurcation. These mean wrestling with the forces of convergence and divergence in chaotic attractors, which we now briefly characterize.

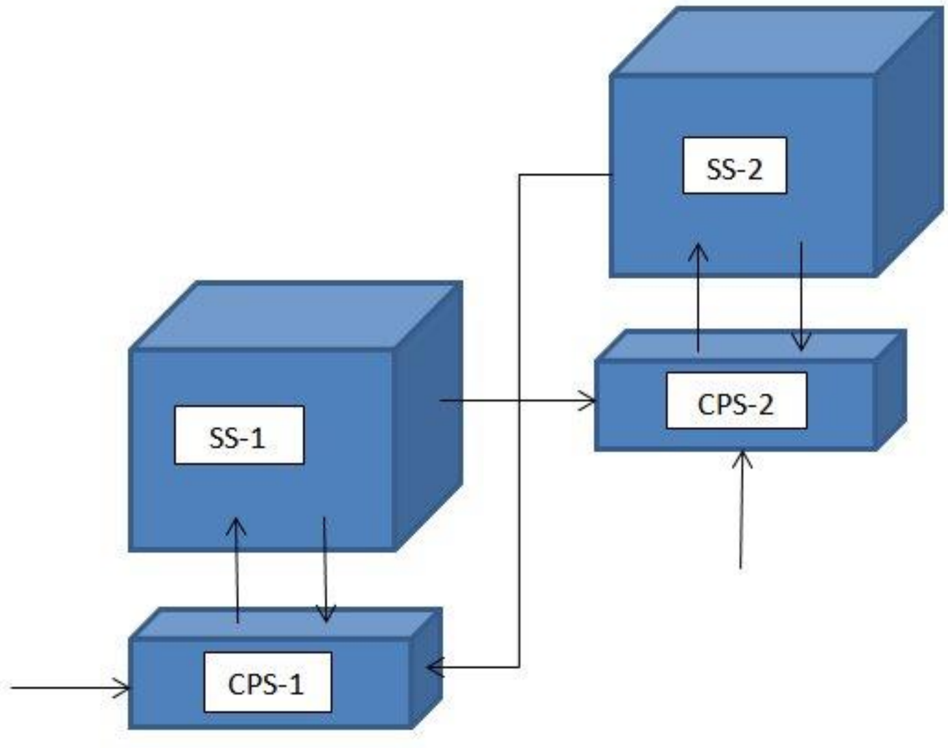


Figure 4. Small Network. SS = State Space, CPS = Control-Parameter Space. Adapted from Abraham & Abraham, 2010.]

You can see two state spaces, each with three variables (could be more or less) and three control parameters (constants) each, and also inputs from each other's state spaces and from the environment. One can have systems with many more nodes. Here self-organization is expressed by the arrows from SS-1 to CPS-1, and SS-2 to CPS-1 and CPS-2

Part IV. Chaos Results from Combining Convergence and Divergence.

We review these properties of chaos that seem most relevant to creativity by examining a very simple chaotic attractor.

"The Rössler attractor exists in a three-dimensional state space and is organized around two saddles. The outset (of index 2 on the x-y plane) of one saddle (red dot of the left image) consists of trajectories that diverge from each other as they spiral outward counterclockwise. The second saddle point (not shown), upper right away from the attractive thick band of the attractor, pulls the trajectories up and turns them back toward the band. Figure 11 shows that the region occupied by a set of trajectories has expanded after one trip around the attractor. (Abraham & Abraham, 2010, p. 15.)

*“The rate or strength of divergence within and convergence to the attractor are given by its **Lyapunov Exponents** (**characteristic exponents** and **multipliers**), one for each dimension, the set of three (one for each dimension in the state space) comprising the **Lyapunov Spectrum**. The largest positive exponent is a measure of divergence between two trajectories for the dimension displaying the greatest divergence.”* (Abraham & Abraham, 2010, p. 18.)

The negative must win or there is no attractor. The greater the divergence, the more the attractor becomes complex, more chaotic. Its fractal dimension, another measure of its complexity, will also increase.

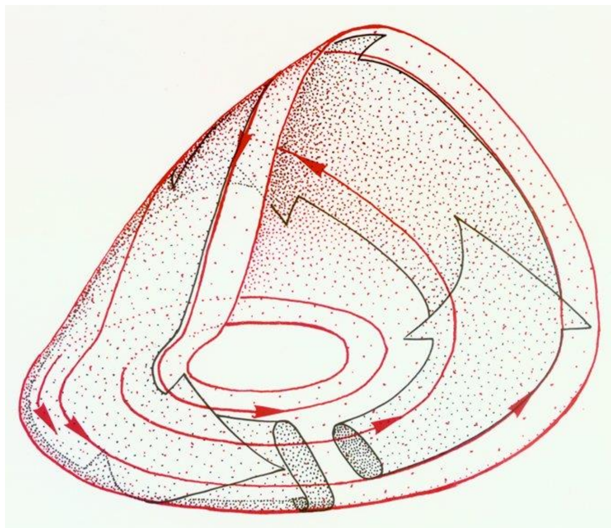


Figure 5. Rössler’s Chaotic Attractor, showing divergence. Image from Abraham & Abraham (2010, p. 16).

The thick attractive surface shows how a bundle of trajectories occupies more real estate after traveling around the attractor once, due to divergence. Convergence brings trajectories from a distance away from the surface toward the surface of the attractor. For this attractor, the vectors of the forces are organized around two saddles, not shown, one near the center of the bottom of the attractor, the other outside to the upper right. Lyapunov exponents express the amount of convergence or divergence there is for each of the dimensions of the attractor. (Letellier & Rössler, 2006).

In discussing Guilford’s (1953) psychometric evaluation of intelligence as requiring convergent thinking (heading for the attractor of the most appropriate solution to a problem or desirable condition), and creativity as requiring divergent thinking (generating multiple solutions), it was conjectured that,

“. . . there is a mixture of divergent and convergent thinking required for creative cognition, and that there is a range of optimal dimensionality to the chaotic nature of this process, in the mid-dimensional range.” (Abraham, 1996, p. 385.)

We could update that on the basis of the edge-of-chaos parameters and the Ueda type explosive bifurcations, noting that if one is moving along a parameter, more and more possibilities are generated, and one sorts through them, dropping some, generating new ones, until there is some satisfaction that some class of solutions or ideas achieves greater stability at the expense of others.

Conclusion

The exploration of concepts from complexity theory is expanding rapidly, and the attempts to develop useful models, research, and technology usually lags behind the development of ever newer generations of our conceptual tools, as does our ability to comprehend them. Whether our conjectures of the potential usefulness of these basic concepts discussed herein prove of any value in understanding the world we live in remains to be seen. We have offered no explicit models of creativity, and have merely suggested a few dynamical and network concepts that could be brought to bear on investigating various aspects of creativity. For a more thorough examination of complexity concepts and their philosophic implications, see Richardson (2010). Meanwhile, we look forward seeing the development of a better understanding of the creative process, so well expressed in Richards (2007). We have focused on the region of instability for complex creative chaotic bifurcations that are smeared in time and distance in state space to which one might stretch the already liberalized phrase, "the edge of chaos".

Dedication

We dedicate this paper to the creative and imaginative pioneers of dynamical thinking in the Winter Chaos Conference and the Blueberry Brain Institute, and our students and colleagues at Silliman University and Saybrook. You will find some of them at: www.blueberry-brain.org and impleximundi.com/tiki-index.php

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End notes

[i] Comments elaborating the some terms and concepts used in Krippner, Richards, & Abraham (submitted) Creativity and Chaos in Waking and Dreaming States.

[ii] From Leibniz's address to the founding meeting of the Society for Chaos Theory in Psychology, Saybrook Institute, 1991: printed as The Leibniz-Abraham Correspondence in F.D. Abraham & A.R. Gilgen (Eds.) (1995), *Chaos Theory in Psychology*. The publisher was kind enough to send complimentary copies to Leibniz, c/o Abraham, who still treasures it.

[iii] Thanks to Ralph Abraham for confirming this interpretation of his and Shaw's diagram of Figure 2. See also Ueda (1980), and for some nice movies of the Duffing forced oscillator, including both trajectories and Poincaré sections, see Kanamaru (2008).

[iv] For the general case of k possible states at each node, $\lambda_c = (1 - 1/k)$

Authors Notes

Frederick David Abraham is the founder of the Blueberry Brain Institute, hidden away in Vermont, where he presides over a brain buckling under the strain of trying to understand the brain and behavior, and how to improve on self and society. He was co-founder of the Winter Conference on Brain Research, The Society for Chaos Theory in Psychology, and the Winter Chaos Conference, and was co-author of two books and several articles. He is also occasional Visiting Professor at Silliman University. Stanley Krippner and Ruth Richards serve as professors at Saybrook University. Correspondence could be sent to frederick.d.abraham@gmail.com.