Symposium on Chaos Theory: Creativity, Waking and Dreaming Life, Psychopathology, and Art by Tobi Zausner, Ruth Richards, Stanley Krippner, & Fred Abraham

APA 2010 San Diego
Comments on Dynamics

Part I. Stability, Instability, and Bifurcations

Supposing you or a patient has some behavioral feature or personality characteristic which you wish to change. Changing such an undesirable attractor we might call a dynamical bifurcation. For me, the most important part of the theory of dynamical systems is that of bifurcation, which means a significant change or transformation. The most common types of attractors are ***static***, ***cyclic***, or ***chaotic***. The most familiar types of bifurcations are ***catastrophic*** (an attractor appears or disappears); ***subtle*** (changing from one type to another); or ***plosive*** (exploding or imploding). We would call persistent behavioral patterns stable, whether maladaptive or healthy. To get from an unhealthy trait to a healthy one requires that one destabilize the system. We can see this by visualizing how dynamics treats such change. One can describe bifurcation properties defining change mathematically or metaphorically, depending the extent to which you wish to formalize the system.

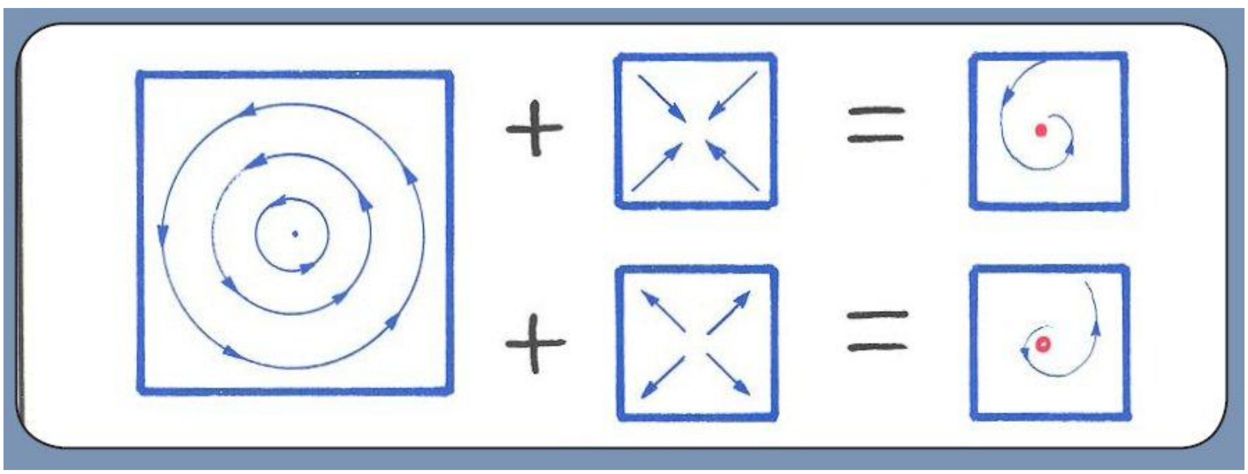

Figure 1. An unstable center can be perturbed into a point attractor or point repellor. Taken from Abraham & Shaw (2005, Fig. 12.1.8, p. 368.)

On the left is a dynamical (aka 'phase' and 'attractor-basin') portrait called a 'center'. The vectorfield of forces at each point in the state space cause the two variables to cycle over the same trajectory through the same starting point. The three cycles start at different points. Starting at a point nearby to those trajectories neither approach nor move away from that trajectory. There is no attractor in this portrait. This model has been used in ecology (prey-predator relationships, Lotka, 1924; Volterra, 1931) and family dynamics (a mother's tension and a son's aggression, Elkaïm *et al*. 1987). The vectorfield and its portrait shown here are extremely unstable. Why? Because the competing forces (say between the variables of the prey and predator populations, or the tension/aggression within the family) are rather equally balanced. Changing the constants amplifying and attenuating those variables, changes the vectorfield, as shown by the vectors in the middle pictures. The upper panel shows the added vectors pointing toward the center of the center, causing trajectories to spiral in to a fixed point attractor, a stable attractor, the portrait at the upper right. If the new vectors point away, a point repellor is created. At the bifurcation point for such ordinary bifurcations, there is no attractor. To get from one attractor to another requires changing one or more constants, with the system losing stability, and passing through the bifurcation point, if a new attractor appears, as the constant is changed further, it becomes more stable again. The phrase 'far from equilibrium' means just this, moving away from stability to instability, and the equilibrium of the effect of changes in the constants of the system.

Thus as claimed, destabilization is an agent of change. But chaos is not usually a feature present at the bifurcation. How can we justify suggesting that it can be. In most human systems, attractors are chaotic, preferably mid-dimensionally, and most bifurcations are between topologically different chaotic attractors. We here conjecture that with complex chaotic attractors, we can have some chaotic features remain, while others in the portrait do, as in the simpler cases, disappear at the bifurcation point. These could participate in the self-organizational capacity of the system to be creative, that is, change. Self-organization involves, especially in intentional systems, navigation in parameter space rather than the state space, but is a function of where the system is in the state space.

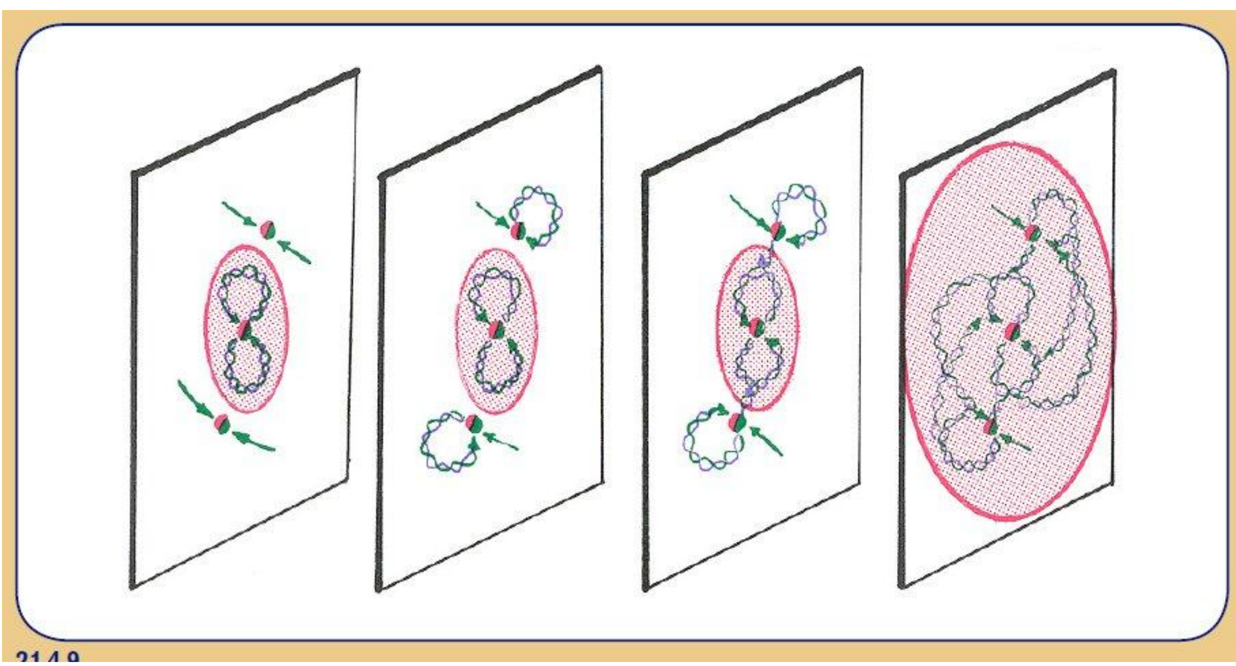

Figure 2. **Portraits of Ueda's Explosive Bifurcation.** Taken from Abraham & Shaw (2005, Fig. 21.4.9, p. 609.)

"Ueda's Chaotic Explosion in 3D (1980). The attractor is in the red area. Outside of it are two saddles with homoclinic connections (from the saddle to itself). The chaotic attractor in the center is a tangle organized around a saddle. Trajectories from the outer saddles are attracted to the center saddle and, along with its own homoclinic connections, form the inset to the center saddle. Then the outset of the central saddle makes connections to the outer saddles, which are then part of the larger chaotic tangled attractor." (Abraham & Shaw, *ibid*.).

You can see why it is called explosive, and see both the former homoclinic, as well as the new, heteroclinic tangles of trajectories. We contend that these chaotic elements contribute to the creative process.

Whether one focuses on creative geniuses or everyday creativity, creativity depends on destabilizing the system. That statement represents, of course, a necessary but not sufficient condition for creativity.

Part II. "The Edge of Chaos"

At first I didn't want to use the phrase 'edge of chaos' because I was convinced that few, including myself, knew what it meant, and that many used it in a very general sense without having any explicit notion of what it meant, outside of the enclave at the Santa Fe Institute, Langdon, Farmer, Crutchfield, Packard, Hraber, and Kauffman *et al*. who developed the ideas. It deals with the patterns of activity in complex adaptive systems (CAS), which are two or more continuous or discrete dynamical systems that are locally interactive by means of their influence on each other's control parameters, more especially those called cellular automata. These are discrete spatial systems (networks, lattices) with global properties much like a dynamical system.

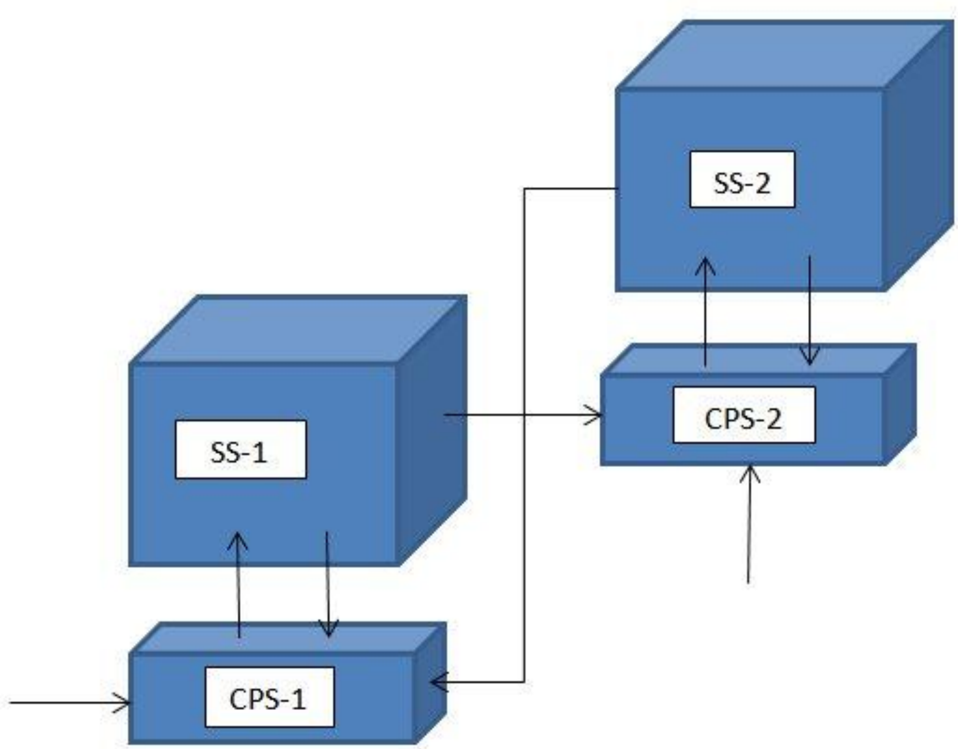

Figure 3. Small Network. SS = State Space, CPS = Control-Parameter Space. Adapted from Abraham & Abraham, 2010.]

You can see two state spaces, each with three variables (could be more or less) and three control parameters (constants) each, and also inputs from each other's state spaces and from the environment. One can have systems with many more nodes.

One such special type of CAS Kauffman called Random Boolean Networks which were discrete (progresses by steps) and all variables were binary.

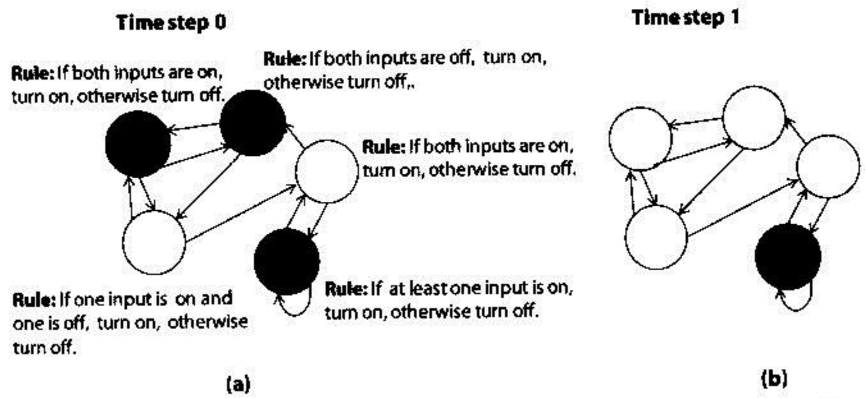

FIGURE 18.2. (a) A random Boolean network with five nodes. The in-degree (K) of each node is equal to 2. At time step 0, each node is in a random initial state: on (black) or off (white). (b) Time step 1 shows the network after each node has updated its state.

Figure 4. Kauffman's KN Boolean network. Taken from Mitchell (2009).

At the left is the initial condition of the network in which each node can take on one of two states (e.g., on or off, 0 or 1, *etc*.), and a set of rules how each node changes when progressing to the next step. The pattern of black and white (e.g., on or off, 0 or 1, *etc*.) nodes change as the system continues through successive steps. Similar networks of cellular automata can have more than two states at each node, and different patterns of inputs from neighbors and rule tables for change at successive steps. The patterns that evolve can take on properties similar to those of dynamical systems, that is, approach static, cyclic, and chaotic attractors, but with such complex dynamics the issue of bifurcation becomes more complicated. Different parameters of the system can affect the evolution of patterns, and can thus produce bifurcation sequences. (See also Malloy *et al*., 2010.)

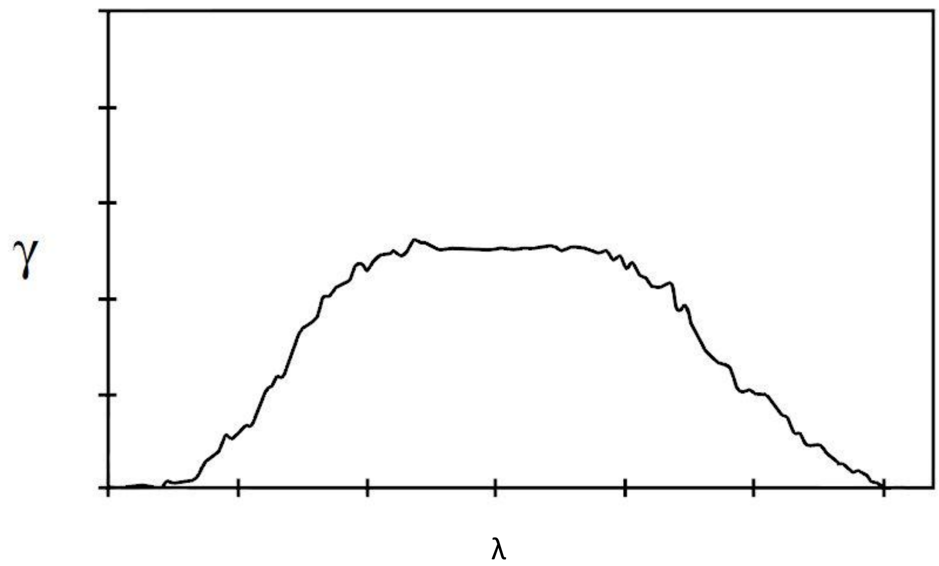

Figure 5. Packard's classic 'Edge of Chaos' experiment, 1988 which shows the results of many runs of simulating such a network.
Legend: γ is the difference-pattern-spreading rate as system evolves, analogous to the measure of divergence in a chaotic dynamical system. λ is the proportion of a specified state nodes can take as specified by the rules of change from step to step.
Adapted by Mitchell *et al*. (1994) from an experiment by Packard (1988).

Packard's network was more complex in having inputs from more neighbors. He measured the diffusion rate λ throughout the system while it evolves as a function a parameter λ that represented the proportion of an arbitrary (named 'quiescent') state in the output of the rule table for the changes of each node at all steps. Thus λ is similar to the Lyapunov exponent, which is a measure of divergence in a chaotic attractor in dynamical systems, a major determinant or measure of its complexity. When λ is low, that would be a fixed point or cyclic attractor, when higher, chaotic. There are two critical regions of γ, one where the curve is rising, one where it falls rapidly. The first gives rise to the term 'edge of chaos'. (Originally it seemed as if some of the authors took chaos to mean so high dimensionally it was random.)

There are four things relevant to us here. (1) This is a region, not a critical bifurcation point as in the classic picture of dynamical systems. Chaos already exists at the onset of this region (the base of the ogive), and thus can be contributing to creativity which we conjecture. Research on cellular automata is an active area of investigation and is revealing many complex things happening in these critical regions. (2) The curve is remarkably like the inverted-U function found in studies of aesthetics and the perception of complexity, suggesting the complexity of cognitive and emotional elements that may be contributing to aesthetic and complexity judgments. (3) Normal mental activity could be driven by a multi-component system. I have previously conjectured a dynamical model in which interactions among mental components may weaken, that is, the coupling constants in their differential equations may be weakening among brain and mental components during dreaming. From the cellular-automata view, there could be a loss of inputs to nodes, and thus a weakening of influence among nodes, enabling more independent local evolution, a less coherent pattern, thus giving it the appearance of random interconnection events in dreaming, but giving creativity a better chance to develop in more localized regions of mental space. (4) Boolean networks have been applied to other psychological investigations, such as perception (Malloy & Jensen, 2008).

Part III. Chaos Results from Combining Convergence and Divergence.

We review these properties of Chaos that seem most relevant to creativity by examining an overly simple chaotic attractor.

"The Rössler attractor exists in a three-dimensional state space and is organized around two saddles. The outset (of index 2 on the x-y plane) of one saddle (red dot of the left image) consists of trajectories that diverge from each other as they spiral outward counterclockwise. The second saddle point (not shown), upper right away from the attractive thick band of the attractor, pulls the trajectories up and turns them back toward the band. Figure 11 shows that the region occupied by a set of trajectories has expanded after one trip around the attractor. (Abraham & Abraham, 2010, p. 15.)

*"The rate or strength of divergence within and convergence to the attractor are given by its **Lyapunov Exponents** (**characteristic exponents** and **multipliers**), one for each dimension, the set of three (one for each dimension in the state space) comprising the **Lyapunov Spectrum**. The largest positive exponent is a measure of divergence between two trajectories for the dimension displaying the greatest divergence."* (Abraham & Abraham, 2010, p. 18.)

The negative must win or there is no attractor. The greater the divergence, the more the attractor becomes complex, more chaotic: its fractal dimension, another measure of its complexity, will also increase.

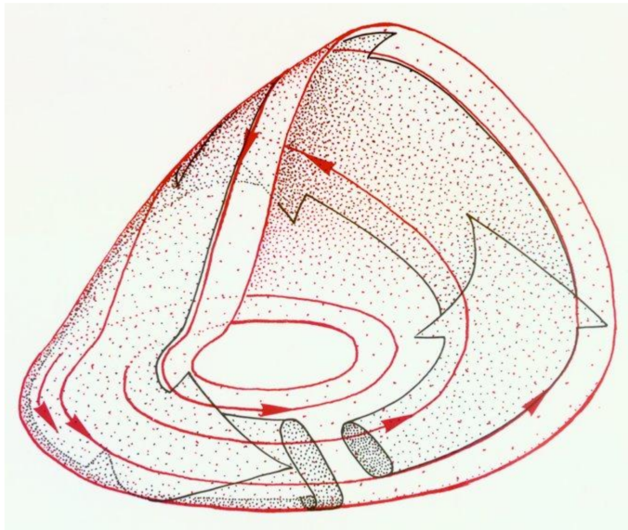

Figure 6. Rössler's Chaotic Attractor, showing divergence. Image from Abraham & Abraham (2010, p. 16).

The thick attractive surface shows how a bundle of trajectories occupies more real estate after traveling around the attractor once, due to divergence. Convergence brings trajectories from away from the surface into the attractor.

In discussing Guilford's psychometric evaluation of intelligence as requiring convergent thinking (heading for the attractor of the single appropriate solution to a problem), and creativity as requiring divergent thinking (generating multiple solutions), I conjectured that,

". . . there is a mixture of divergent and convergent thinking required for creative cognition, and that there is a range of optimal dimensionality to the chaotic nature of this process, in the mid-dimensional range." (Abraham, 1996, p. 300.)

We could update that on the basis of Packard's result (inverted-U of figure 5), noting that if one is moving from left to right in climbing the parametric ladder of number of relevant solutions, more and more possibilities are generated, and one remains there sorting through them, dropping some, generating new ones, until there is some satisfaction that some class of solutions, or solution becomes dominant, and increasing λ, makes the complexity, γ, drop off rapidly to the lower dimension of the appropriate solution.

The Santa Fe brain trust is investigating computational networks, which considering the plethora of modeling possibilities, would have to be considered still in their infancy, but very promising to a wide range of biological, psychological, and cultural issues. They suggest that they may reveal complexities that we can add to our own modeling of psychological activity, especially in combining the relationship between dynamics and networks that can save us from some of the oversimplifications with which we currently struggle. Replacing the exactness of a bifurcation point with the smeared bifurcation region is an example, along with the nature of the underlying psychological processes involved. For a more thorough examination of the CA approach and its philosophical implications, see Richardson (2010).

References

Abraham, F.D. (1996). The dynamics of creativity and the courage to be. In W. Sulis & A. Combs (eds.), *Nonliner dynamics in human behavior*. Singapore: World Scientific.

Abraham, F.D. (2007). Cyborgs, Cyberspace, Cybersexuality: The Evolution of Everyday Creativity. In R. Richards (Ed.), *Everyday Creativity and New Views of Human Nature: Psychological, Social, and Spiritual Perspectives*. Washingto, D.C.: American Psychological Association Uptdated versions at: http://www.blueberry-brain.org/chaosophy/Cybersexuality/Cybersexuality-creativity-bbi-v6a2.htm

Abraham, F.D., & Abraham, R.H. (2010). A very very brief visual introduction to the theory of dynamical systems. Invited chapter for a forthcoming book by Hector Sabelli. Now parked at: http://www.blueberry-brain.org/dynamics/A%20Very%20Very%20Brief%20VisIntroTDS.pdf
[Figures for slides 1 & 2 can also be found here.]

Abraham, R.H., & Shaw, C.D. (1982-2005). Dynamics: *The geometry of behavior*, *Part I: Periodic Behavior*. Santa Cruz: Aerial Press. *Parts II, Chaotic Behavior*, 1984, *III, Global Behavior*, 1985, & *IV, Bifurcation Behavior*, 1988, followed. All four volumes were collected into a second edition, *Dynamics: The Geometry of Behavior*, 1992, Redwood City: Addison-Wesley. Now (2005) available as a CD.

Elkaïm, M., & Goldbetter, Goldbeter, A., & Goldgeter-Merinfeld. E. (1987). Analysis of the dynamics of a family system in terms of bifurcations. *Journal of social and biological structures*, *10*, 21-36.

Kauffman, S.A (1995). *At home in the universe: The search for laws of self-organization and complexity*. Oxford: Oxford University Press. [See also http://pespmc1.vub.ac.be/BOOLNETW.html]

Langton, C.G. (1990).Computation at the edge of chaos: Phase transitions and emergent computation. *Physica D*, *42*, 12-37.

Lotka, A.J. (1924). *Elements of physical biology*. Reprinted as *Elements of mathematical biology*, 1956, New Youor: Dover.

Malloy, T.E., Butner, J., Dickerson , C. & Cooper, J.M. (2010). Fearless-evolution on Boolean landscapes: Boolean phase portraits reveal a new navigation strategy based on fearful symmetry. *Emergence: Complexity & Organization*, accepted for publication in issue *12(3).*

Malloy, T.E., & Jensen, G.C. (2008). Dynamic constancy as a basis for perceptual hierarchies. Nonlinear Dynamics, Psychology, and Life Sciences, 12, 191-203.

Mitchell, M. (2009). *Complexity: A guided tour*. New York: Oxford.

Mitchell, M., Crutchfield, J.P., & Hraber. (1994). Dynamics, computation, and the “Edge of Chaos”: A reexamination. In G. Cowan, D. Pines, and D. Melzner (editors), Complexity: Metaphors, models, and reality. *Santa Fe Institute Stuides in the Sciences of Complexity, Proceedings, 19*. Reading: Addison-Wesley. Parked at http://web.cecs.pdx.edu/~mm/dyn-comp-edge.pdf

Packard, N.H. (1988). Adaptation toward the edge of chaos. In J. A. S. Kelso, A. J. Mandell, and M. F. Shlesinger (eds.), Dynamic patterns in complex systems, 293-301, Singapore: World Scientific.

Richardson, K.A. (2010). *Thinking about complexity: Grasping the continuum through criticism and pluralism*. Litchfield Park: Emergent Publications. ISBN: 978-0-9842164-5-1

Ueda, Y. (1980). Explosion of strange attractors exhibited by Duffing’s equation. *Nonlinear Dynamics*, R..H.G. Helleman (ed.), pp. 422-434. NewYork: New York Academy of Sciences.

Volterra, V. (1931). *Leçons sur la théorie mathématique de la lutte pour la vie*. Paris: Gauthier-Villars.