Eve in the Garden of Chaos

Frederick David Abraham[1] ©2016[2]

The greatest value of a picture is when it forces us to notice what we never expected to see.

John Tukey, 1977

Prologue

Complexity theories include a number of related approaches to studying the nature of the evolution of complex integrative processes. My favorites are dynamical systems theory and network theory because they give me a clear conceptual view of important integrative processes, especially those of the roles of self-organization and stability/instability upon which the control of bifurcation depend. Also because they both emphasize the visualization of mathematical processes. Dynamics is especially useful in dealing with systems whose variables and components are continuous, such as variables in differential equations, or in real systems, such as electrical and biochemical activity in nervous systems. Network theory is applicable where variables are discrete, such as variables in difference equations, or in real systems, such as genes in a genetic system, or individuals in a social system. But these approaches can supplement each other in important ways. For example, simple dynamical systems as well as their variables can be nodes in a network. And conversely, network statistics can be fed into dynamical statistical methods.

Complex systems usually involve the temporally nonlinear interaction among different factors or variables. Nonlinear simply means that a mathematical description of their interactive iteration involves multiplicative, exponential, logarithmic, or trigonometric terms. This property has to be present in order to enable the most important aspects of complex systems, namely bifurcations and chaos. Most systems studied by biological and human sciences exhibit these characteristics.

Both dynamical and network theories have been applied to just about every scientific and cultural process. Examples include ecology, neuroscience, chemical reactions, gene regulation, semantics, anthropology, paleontology, transportation, metabolic pathways, organizations, telecommunications, electrical grids and circuits, sociology, cosmology, economics, World Wide Web, authorship and disciplinary connections, *etc*.

Integrative science and thinking have moved into a new era of complex modelling and data handling, accelerated all the more since Innes and McLuhan highlighted issues of hyper-communication. Science tries to understand how things work. Scientific success was established by careful observation while eliminating the influence of variables on the system under study, e.g., Galileo. But simple natural laws ignore the richness of the real interactions that go on in the world, e.g., simple laws of operant conditioning ignore the phenomenology that goes into those apparently simple behaviors. Simple laws have been of great help in the real world, but we now feel that we can make more penetrating understanding and practical uses by studying more complex systems, e.g., in the fields of prosthetics and psychotherapy.

[1] frederick.d.abraham@gmail.com, www.blueberry-brain.org, Blueberry Brain Institute, Waterbury Center, VT, USA.

But there is the difficulty that our analysis of systems can be too complex. Korzybsky's famous dictum, "The map is not the territory" cautions not only that the map leaves something out, but that if the map were to include everything, it would be as complex as the territory, and thus stress our cognitive capacities just as much as the real territory. So my preference is for a balance of simplicity and complexity (see Cohen & Stewart, 1995; Berthoz, 2012).

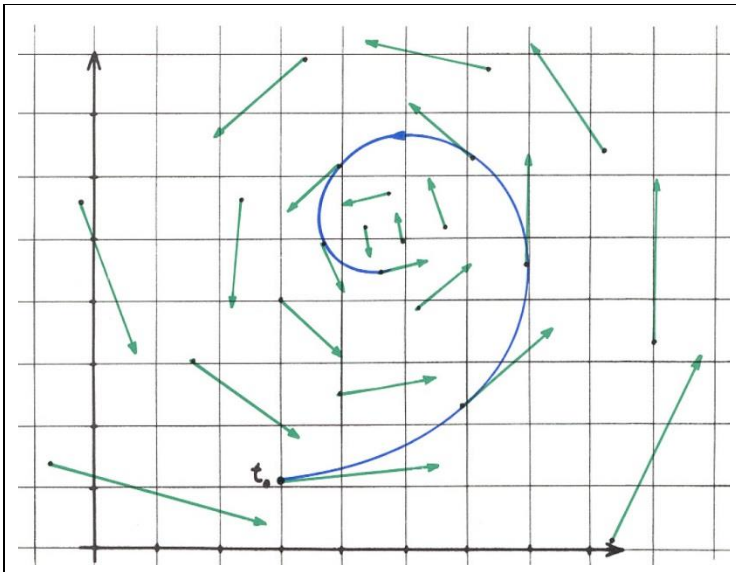
A final comment on my brief personal basics of the philosophy of science is to note that the history of Philosophy dotes on the separation of ontology and epistemology, a false separation as far as I am concerned since they are entangled in phenomenology. Similarly, I think modelling and operational observation in science are inseparable phenomenologically. Dynamical theorizing emphasizes the importance of contextual factors giving meaning to our ideas and findings, and so it is with science itself (Bridgman, 1936; Chang, 2009). Gödel's incompleteness, if you will.

Basic Features of Dynamical Systems

A ***dynamical system*** is comprised of a set of variables changing while interacting over time.   If their behavior is sufficiently well-behaved, they can be represented by graphs and equations, such as ordinary differential or difference equations which may contain nonlinear terms

State Space, Vectorfield, & Trajectory

The ***state space*** is comprised of a set of points within a Cartesian space defined by the variables of the system. Each point has a velocity vector describing its desire to change. The ***vectorfield*** is comprised of those vectors. The state space and vectorfield may be represented graphically and/or with the notation of vector calculus.



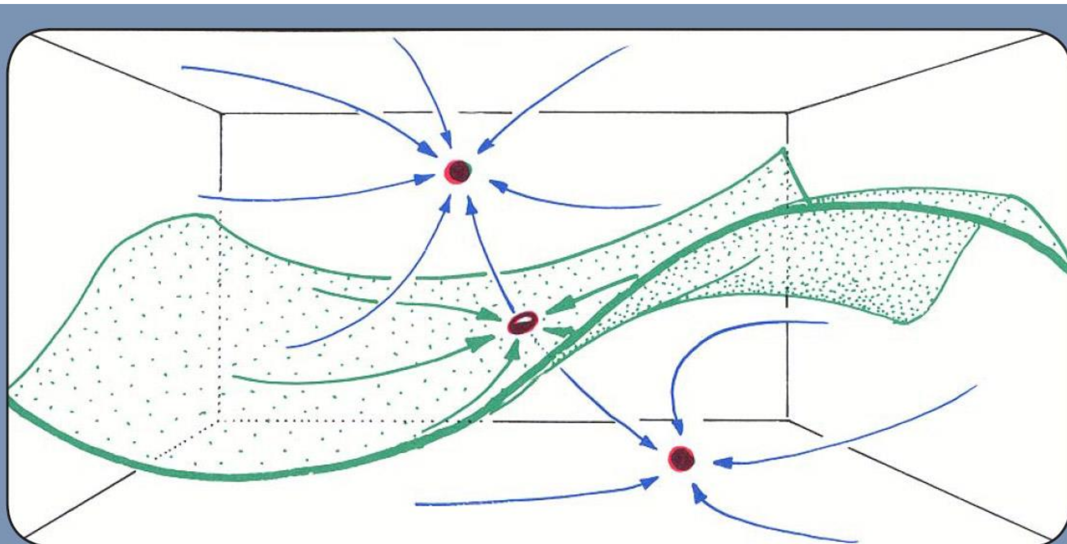
State Space shown with a few vectors of the vectorfield and the trajectory they generate starting from one point, t_o.

Abraham Shaw, 2005, Part I, Fig. 1.2.9, p. 25[3]

A ***trajectory*** represents the evolution of the state of the system which is propelled by a sequence of vectors, the directed 'forces' of the system. This process can be represented mathematically by the procedure of integration in vector calculus. Conversely, if you have a given trajectory, its vectors can be revealed by differentiation. Thus, the velocity of the trajectory need not be constant, which can be revealed by the vectors at each point. Time can be visualized with colorization or hatch marks along the trajectory and comparing differences in distances made at equal time intervals. The variables of the system can also be represented by time series, one for each variable; the trajectory in state space is thus a combined version of them

Attractor Basin Portraits

The state space does not have an axis for time, but rather time is contained within the vectors and trajectories. The state space may be filled with these, and in so doing reveals many patterns, which graphically tell the story of the dynamics of the system, and exhibit some from among many critical features. These have been called state space portraits, phase space portraits, portraits, or, my favorite, ***attractor-basin portraits*** (Hanson & Crutchfield, 1992). Here is an example showing some critical features often found in such portraits.[3]

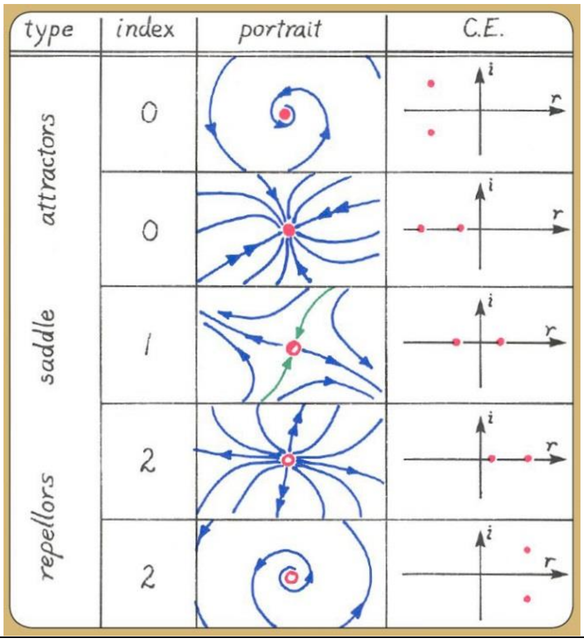


10.1.5.
In this portrait of a simple bistable system in 3D, there are again two attractors. Both are rest points. Their basins are three-dimensional, and are bounded by a surface. This surface, the separatrix in this example, is the inset of a saddle point of index 1.

There are three critical points, two are ***fixed point attractors*** (stable), to which trajectories only tend. The one in between is a ***saddle point***, from which trajectories depart toward the fixed point attractors. Other trajectories approach it, forming the curved sheet that defines the ***separatrix*** between the ***basins of attraction***. "Index" in the caption is the dimension of the ***outset***. There are similar features for cyclic attractors and saddles as well, as shown in the next figure.[3] A ***repellor*** is a critical feature from which all trajectories depart. An ***inset*** is comprised of all nearby trajectories that approach a critical feature. critical feature

[3] Abraham & Shaw, 2005, Part III, Section 1, p. 341, 342.

Quantifying Properties of Attractor Basin Portraits



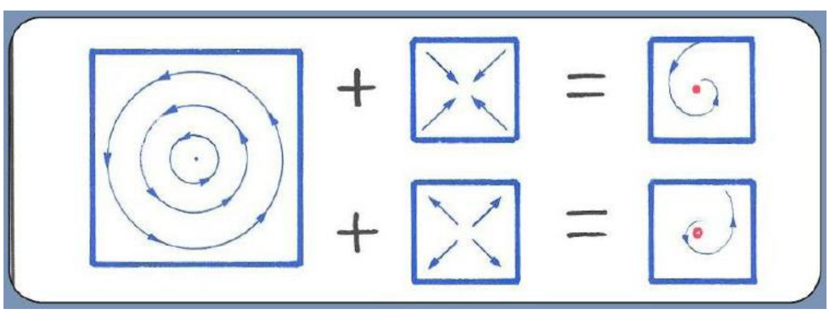
Characteristic Exponents (aka eigenvalues) and Indices for Typical Limit Points in Two-dimensional State Spaces.[4]

The characteristic exponent (CE) of a critical point is a complex number whose real component represents the rate of approach and departure of the trajectories, while the imaginary component represents the rate of rotation of the trajectories. These points are hyperbolic, that is, the real component is not equal to 0. The index is the dimension of the outset of a critical point.

Such measures can also be made for 1 to n dimensional Cartesian (embedding) state spaces, and to cyclic attractors, saddles, and repellors.

Stability

A stable system is not greatly perturbed by small changes in a system's parameters, and conversely an unstable system is significantly changed by such a change. These can be viewed topologically as a change to the ***Vectorfield*** and consequently, its trajectories, and its attractor-basin portrait.



This portrait[5] shows an unstable ***center*** (at left) where trajectories cycle continually through their initial point; the trajectories neither attract nor repel. It lies at the bifurcation point between attractive and

[4] Figure 6.4.8 is from Abraham & Shaw, 2005, Part II, p. 227. Figures for the CE's are Argand diagrams. "The geometric representation of complex numbers [***Argand diagram***] was championed in 1806 by Jean-Robert Argand (1768-1822), a French bookkeeper and amateur mathematician." From Sprott, 2003, p. 75.

repelling fixed points. It is easily perturbed to a point attractor or repellor by slight additions to the vectorfield which are also reflected in the control parameters. The system is more stable the further the control parameter is from the unstable bifurcation point.

Chaos

Here is an image of the iconic chaotic ('strange') Rössler Attractor (Rössler, 1976) showing some of its properties.

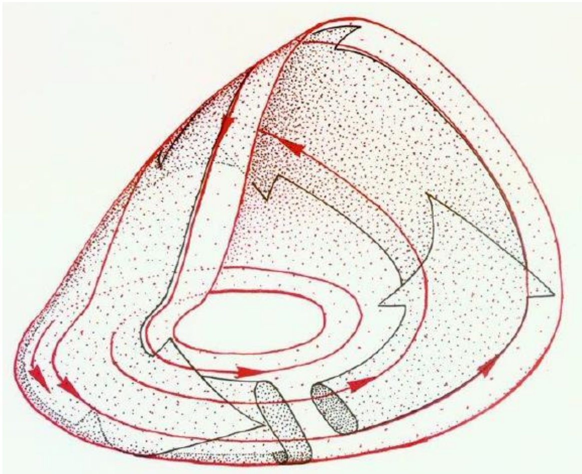


Figure R(a). Expansion of a set of Trajectories in one turn around the attractor.[6].

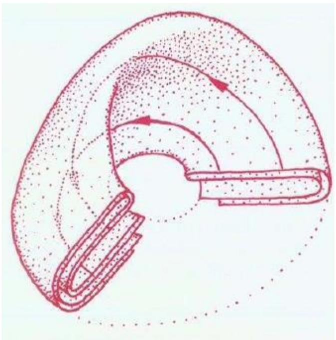


Figure R(b). Fractal Microstructure.[7].
Figure 12a (above). *Poincaré Sections.*
Perpendicular to the Rössler Band

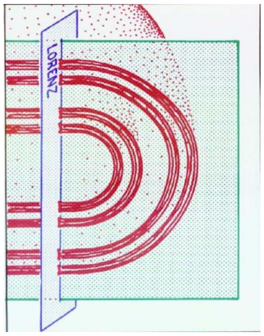


Figure R(c). *Lorenz Section*
Perpendicular to the Poincaré Section.

[5] Figure 12.1.6 is from Abraham & Shaw, 2005, Part III, p.367.

[6] Abraham & Shaw, 1984, cover and, 2005, Part II, similar to Fig. 9.1.9, p. 302.

[7] Abraham & Shaw, 1992, pp. 319-320, and in 2005, Part II, Figs. 9.4.4, 9.4.5, p. 319, and Figs. 9.4.6 and 9.4.7, p. 320.

Figure R(d). The *Lorenz Section* reveals the pattern of the layering of the Rössler Attractor.

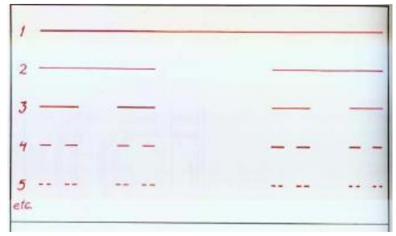


Figure R(e). The *Cantor Process*. Successive steps of decimating a line illustrate the self- similarity across scale evident in the chaotic attractor. Middle-thirds decimation shown.

One of its renowned properties evident in Figure R(a) is the short term divergence of trajectories from nearby arbitrary initial points, the so-called "sensitivity to initial conditions" and the basis for loss of predictability for future positions, given the lack of precision and its replicability of the initial position. Notice also, that separated points seem to converge. Thus there is both divergence and convergence in a chaotic attractor, the trajectory always want to turn back to the invariant manifold, i.e., the attractor. The rate of divergence is measured by characteristic multipliers, or Lyapunov exponents, one for each variable, thus comprise a vector called the Lyapunov Spectrum. A positive exponent measures divergence, a negative one, convergence. The sum of the positive exponents must be greater than the sum of absolute values of the negative exponents; otherwise the trajectory would flee the space. The spectrum may be used to characterize both theoretical and empirical trajectories. Often, authors only give the largest positive exponent, but I don't like that loss of information.[8]

The attractor is embedded in a Cartesian space of integer dimension equal to the number of variables in the system. The attractor does not fill that space, and the extent to which it does is estimated by the ***correlation dimension*** or ***fractal dimension***, as it appears to be like a fractal (Figures R(b) through R(e)). There are a couple of approaches used to measure it. One needs only a single variable, and creates more by making several copies of it, and time delaying each successive copy from the preceding one. At each step, the attractor is graphed and a fractal dimension computed on it. The process is repeated until the attractor stops 'plumping up' and the fractal dimension hits asymptote. This procedure can be used with both theoretical and empirical time series, and can be fed other variables in a multivariate system, rather than time delayed time series. A generalized spectrum of fractal dimensions is generated by a systematic change in a parameter of the computation.[9] Enhanced extensions of these methods include singular value decomposition and false nearest neighbors.[10]

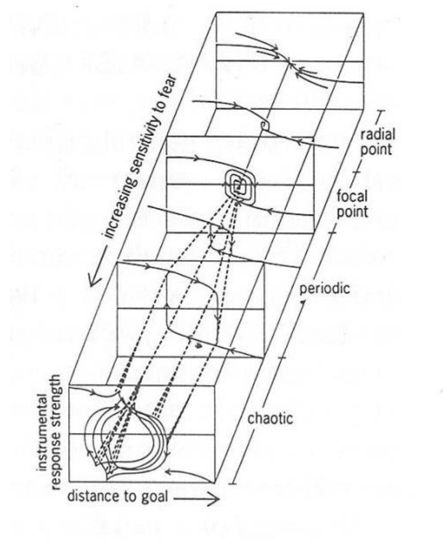
[8] Lyapunov Exponents: Abarbanel, 2012; R. Abraham & C. Shaw, 2005; Kantz & Schreiber, 2004; Ott *et al*., 1994; Sprott, 2003.

[9] Fractal dimension: F. Abraham, 1997; 2009; Grassberger & Procaccia, 1983a,b; Sauer et al., 1991; Generalized fractal dimensions: Grassberger, 1983; Hentschel & Procaccia, 1983; Paladin & Vulpiani, 1987; Solé & Manrubia, 1995; Sprott, 2003.

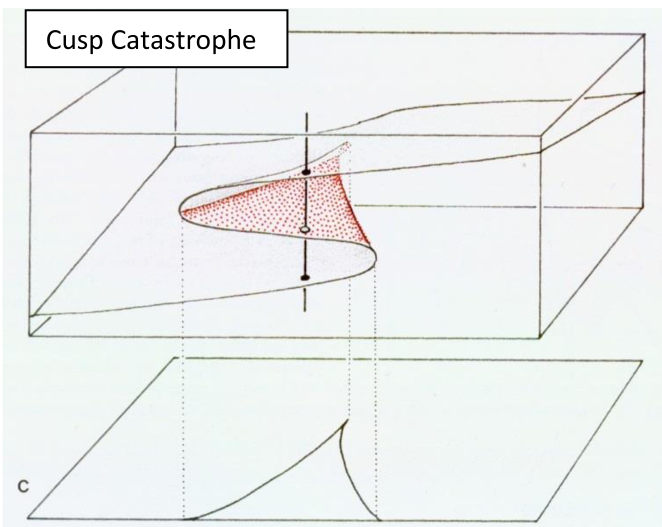
[10] Singular value decomposition: Albano *et al*., 1988; Broomhead & King, 1986; Ott *et al*., 1994. False nearest neighbors: Abarbanel *et al*., 1992; Boker & Bertenthal, 1995; Kennel *et al*., 1992; Stewart, 1996.

Bifurcations and their Portraits[11]

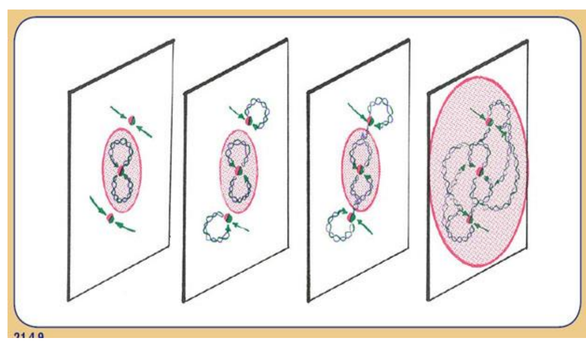
Attractor-basin portraits may vary as system parameters are changed. ***Bifurcation portraits*** are especially useful to depict these changes. ***Bifurcations*** are created when changes in the magnitude, the relative strengths of influence of a system's variables, takes it over some value(s) of one or more of their control parameters, the bifurcation point. Thus, the bifurcation portrait is the representation of the attractor-basin portrait as a function of control parameters of the system. There are many ways to show this. Here is an imaginary one for the classic Miller-Dollard approach-avoidance conflict (Miller, 1959). Figure adapted from Abraham (1993) and Abraham, Abraham, & Shaw, 1990, cover (1990). It follows the classic point–>cyclic–>chaos bifurcation sequence of subtle bifurcations.



For generic systems, there are three basic types of bifurcations, ***subtle***, ***catastrophic***, and ***plosive***. ***Subtle bifurcations*** have topologically different attractors on either side of the bifurcation point) as seen in the above figure (technically there is no attractor at the bifurcation point).



Ueda Explosion[12]



Three homoclinically tangled saddles merge to explode into a single attractor of heteroclinically tangled saddles.[13]

[11] They are also known as bifurcation diagrams or response diagrams.

[12] Ueda, 1980.

[13] Figure from: Abraham & Shaw, 1988, p. 172; 1992, p. 609.

A ***catastrophic bifurcation*** involves the appearance or disappearance of an attractor. The famous ***Cusp Catastrophe*** is usually shown in a 3D ***bifurcation portrait*** (upper left figure), where the horizontal plane represents two control parameters, and the vertical axis represents the 1D state space for the variable of the system. It has been used to depict many aspects of human and social behaviors among others. The curved surface represents the loci of the point attractors (two in the area of the folded surface and repellor (exists only in the folded area). Entry into or departure from the folded area (projected on lower left figure) by means of varying control parameters determines the bifurcations.

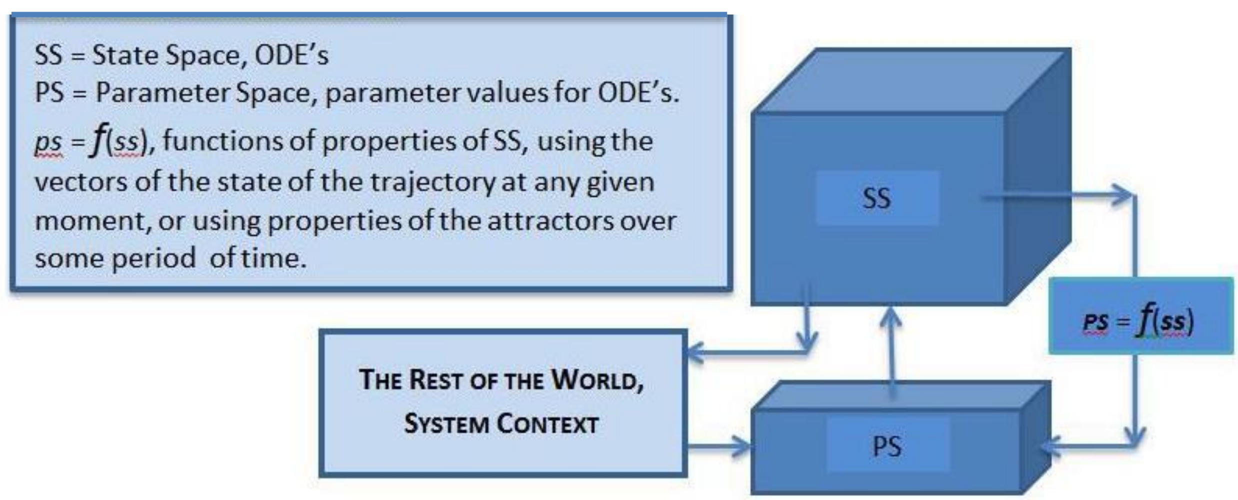
A ***plosive bifurcation*** involves and explosion or implosion in size of an attractor due to a collision of an attractor with an unstable feature of the portrait, such as a saddle. Ueda was the first to describe one.[12] Another well known example is the Blue Sky Catastrophe.[14]

I wish to make a couple of points now, but reserve more discussion until after taking up networks more fully. Most systems we find interesting in psychological and social, and many biological systems are almost always operating in chaotic processes, albeit in varying degrees of dimensionality, most bifurcations are chaos-to-chaos ones. They often tend to occur in cascades while passing through a series of bifurcation points within small or micro regions of parameter space, a macro bifurcational process I have called ***µ-bifurcation*** (Abraham, 2016b).

Self-Organization, Intentional Systems, and Control of Chaos

Self-organization refers to control parameters being a function of the state of the system; the system is changing its own behavior.[15] The changes can be gradual, or bifurcational.

Figure 16. Self-Organization



Multiple dynamical systems can be interconnected into ***complex dynamical systems***. ***Complexity Theory*** may be consider as a superset of Systems Theory and may extend some of the basic ideas to hybrid networks that may include continuous and discrete systems. There are many varieties of these, but the most generalized and versatile of these include ***complex adaptive systems***, ***agent-based systems,*** **and** ***scale free networks***.

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14 A good summary is in Thompson & Stewart (1986, 268-272) which also gives references to Abraham & Simó, Abraham & Stewart, Zeeman, and Diener

15 Figures 16, 17, & 18 are taken from Abraham, 2015a.

Here (infra) is a representation of a simple complex network, comprised of two dynamical systems. In our scheme***, self-organization*** is simple conceptually, but allows for complex outcomes. Self-organization can thus occur not only by influencing its own control-parameters (we could call that ***homoclinic self-organization***), but also mediated via its influence on other components in the network (we might call that ***heteroclinic self-organization***).

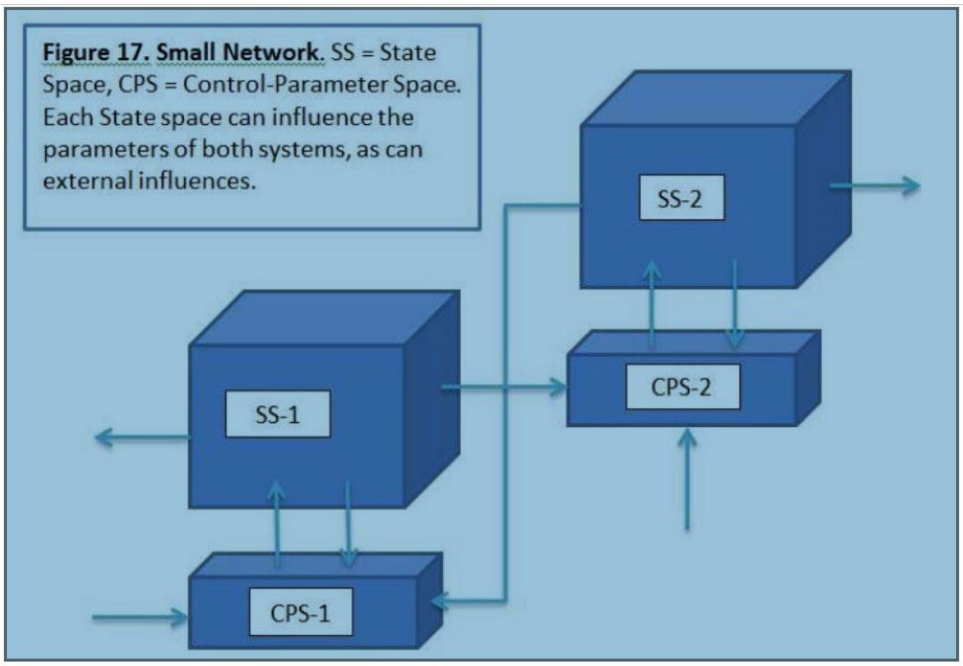

Figure 17. Small Network. SS = State Space, CPS = Control-Parameter Space. Each State space can influence the parameters of both systems, as can external influences.

This definition of self-organization is based on that of Abraham and Shaw (1982) and fits Ashby's (1962). It differs from his in that it allows exploration of parameter space to investigate the various behaviors of which a system is capable without necessarily directing the system to a good or bad organization, leaving such decisions to *a posterior* evaluations. Having said that this exploratory empirical capability of self-organization differs from Ashby's "good" result (there can be bad results as well, thus the need for therapy), it is to be admitted that we hope self-organization will operate to benefit the system, which leads us to a consideration of intention and control of chaos.

Intentional systems can navigate in parameter and state space (Abraham, 1994a; Freeman, 2012); Kugler *et al*., 1990, 1991). They can make choices among future attractors. That is, such systems can make choices among future attractors. This would seem to imply a self-awareness or learning of at least some aspects of both its state and parameter space, and maybe some of the whole network, and perhaps of the network's context. Please enjoy the paradox, to be visited later. Emergent properties of the network can flow from these self-organizational features. ***Emergence*** simply means we, or nature, has not yet explored the whole of the parameter space before, so we see system behaviors being realized for the first time, or as Goldstein (1999) puts it, "during the process of self-organization". Navigation can be very explicit as with a dynamical model, or exploratory, unsure of exactly what it seeks, if anything.

Control of Chaos refers to the modification of the trajectory of a chaotic system in real time. While a given chaotic attractor could be optimal for such a system, local variations could put some portions of the attractor out of its comfort zone, so an active feedback is necessary to move the trajectory back into an area that will keep the trajectory from going to such areas. The system could be modified thusly:

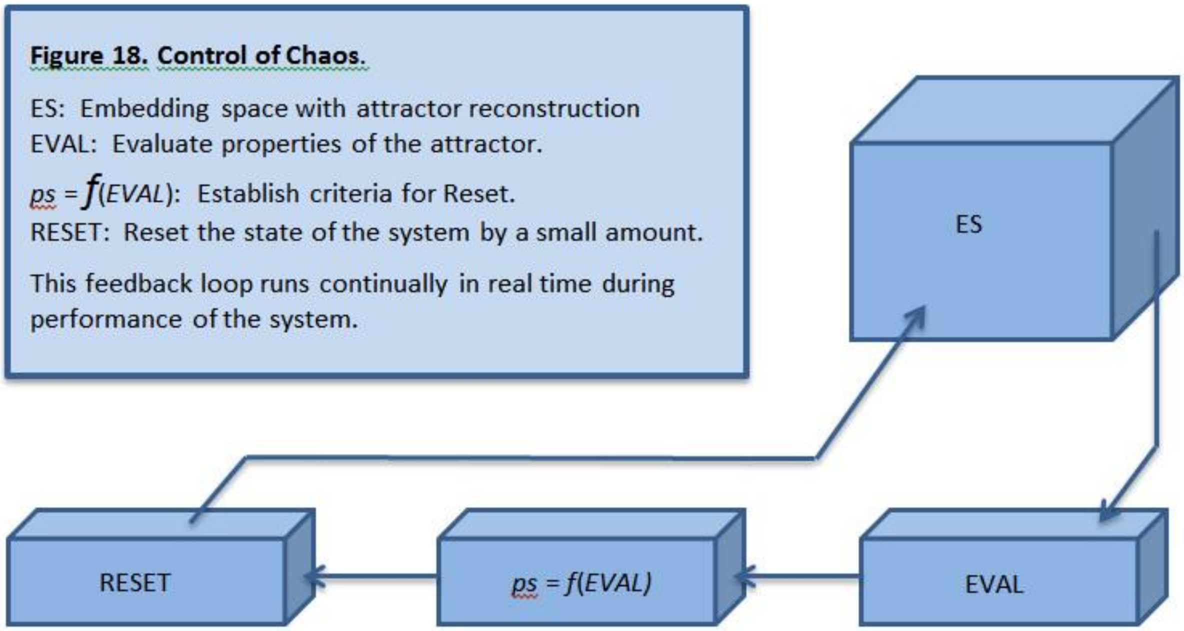

Figure 18. Control of Chaos.

ES: Embedding space with attractor reconstruction
EVAL: Evaluate properties of the attractor.
$ps = f(EVAL)$: Establish criteria for Reset.
RESET: Reset the state of the system by a small amount.

This feedback loop runs continually in real time during performance of the system.

Most of the theoretical developments have been concerned with taming unruly chaotic behavior to more stable chaotic behavior, to regimes of periodic or near periodic behavior, or to fixed point behavior. (Boccaletti *et al*., 2000; Li et al., 2016; Ott, 2006; Ott *et al*., 1990; 1994). The system may or may not be considered intentional, but may, of course, reflect the intention of the designer, if there be one (Ashby's 'good organization', 1962). Garfinkel *et al*. (1992) provide an interesting instantiation with an in vitro cardiac preparation using recording, stimulation, and online analysis of evolving Pointcaré maps of the attractor. This could lead to more nuanced prosthetic enhancements to pacemakers that can reset rhythms upon earlier detection of a trajectory entering dangerous territory of the state space.

Chaotic attractors may be (but are not necessarily) beneficial to many biological, social, ecological, psychological systems, as well as physical and mechanical systems, the motivation for this volume, and this control process may comprise one strategy for 'therapy' of such systems. On the other hand, as Sprott (2003, p. 146) has put it: "Sometimes it is desirable to use feedback to keep the orbits unstable ('anti-control' of chaos.)"We might further suggest that optimizing ***to*** chaotic regimes of mid-dimensional complexity makes for more creative and evolutionary processes that involve both convergent and divergent tendencies (Alford & Abraham, 2011; Guilford, 1953; Krippner *et al*., 2014; Richards, 2000, 2001, 2007; Schuldberg, 2007). For a very complex attractor, see Li *et al*, 2016.

Basic Features of Networks

Before turning to networks of greatest interest to psychology and the life and social sciences, it might be interesting to characterize some similarities of continuous to binary networks[16].

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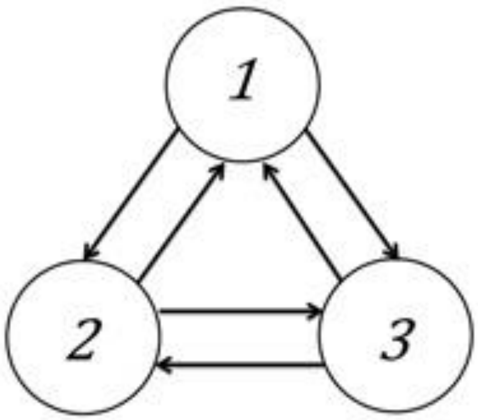
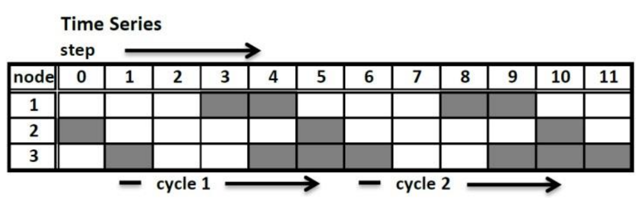
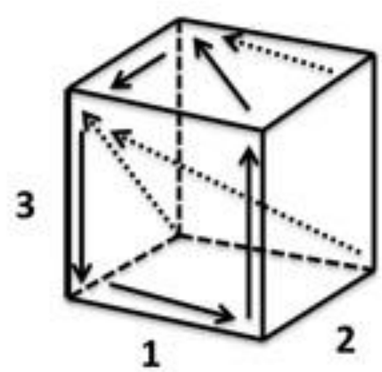
[16] Binary networks often are ***cellular automata***, which are comprised of a homogeneous (in lattices of grids or rings) set of nodes, each of which has the same set of logical rules for connections to its neighbors. Can you change the logical rules for the network shown below to make it a cellular automaton? Cellular automata were introduced by [Ulam and] von Neumann (1948).

Similarities to Continuous Dynamics

The network diagram below shows a network of three binary variables and their connections, some logical rules for their change of state at each iteration, the three time series (rows) for each variable, and their attractor portrait. Replace the binary variable with variables x,y,z of the Rössler system, their differential equation for the logical rules, and the continuous Rössler Attractor for the discontinuous one for the binary network. It can be a matter of taste and intuition to choose one or both representations for any given system, each can complement insights from the alternative one.

	node 1 = 1 if 2 ^ 3 = 0 node 2 = 1 if 1 ^ 3 = 1 node 3 = 1 if 1 v 2 = 1
System Diagram: Variables (Nodes) 1,2,3	The logical rules determining states at step *n* given the states at step *n*-1
	
Time series for the three nodes being synchronously stepped for 11 steps after arbitrarily choosing an initial state from among the eight possible at time step 0. It has a 5-cycle attractor, the first two cycles being indicated. The trajectory for this time series starts from the right rear lower vertex in the state space shown in the next panel. Gray is 'on' or 1; white is 'off' or 0.	All possible trajectories in 3D state space. Right, back, and up represent the 'on' states for nodes 1, 2, 3. The solid arrows belong to the cyclic attractor; the dotted arrows are the single step approaches to the periodic attractor for the three initial conditions not on the attractive cycle.

Note that after zero or one step the attractor is periodic, and has to evolve similarly to a point or cyclic attractor for any number of nodes. Thus chaos ultimately must be a transitional feature found in a periodic trajectory for which we and our computers are just not sufficiently patient to discover, as its state space is 2^n for *n* nodes (Mitchell, 2009; Kauffman, 1995). Kauffman (1994, loc. 1058) notes that basins of small cyclic attractors can trap advantageous autocatalytic networks, but I have conjectured, that the more ubiquitous chaotic attractors might serve similar function (Abraham, 2015b). Attractor-basin portraits can simultaneously exhibit point, cyclic, and chaotic attractors (Li & Sprott, 2014, 2016; Sprott *et al*. (2013). These attractors exist in networks as well as continuous systems.

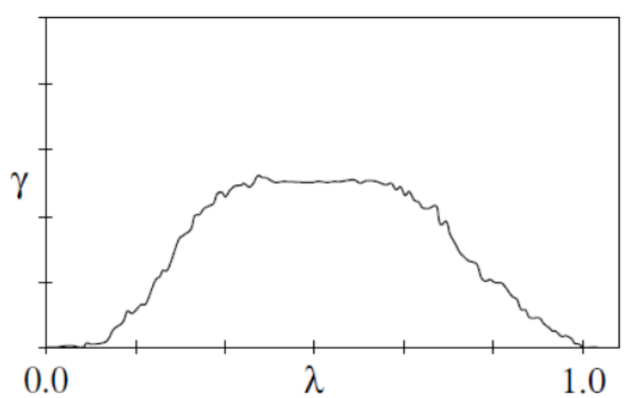
Measures of Complexity in Attractors within Cellular Automata (Li *et al*., 1990).

1. ***Spreading rate***, γ, describes how trajectories for nodes within a neighborhood become increasingly different over time. These can be averaged over the whole space, or used to define attractive basins, with local and global properties for each attractor.
2. ***Entropy***, $S = -\Sigma\, p_i \log(p_i)$ also often designated as
3. ***Mutual information***, *M*, measures the correlation, that is, similarity of series in space-time. $M = \Sigma\Sigma p_{ij} \log(p_{ij}/p_i p_j)$

These are comparable to characteristic exponents, mainly Lyapunov's positive exponents, fractal dimension, and information measures for continuous dynamics.

The 'Edge of Chaos' yields to the μ–Bifurcation

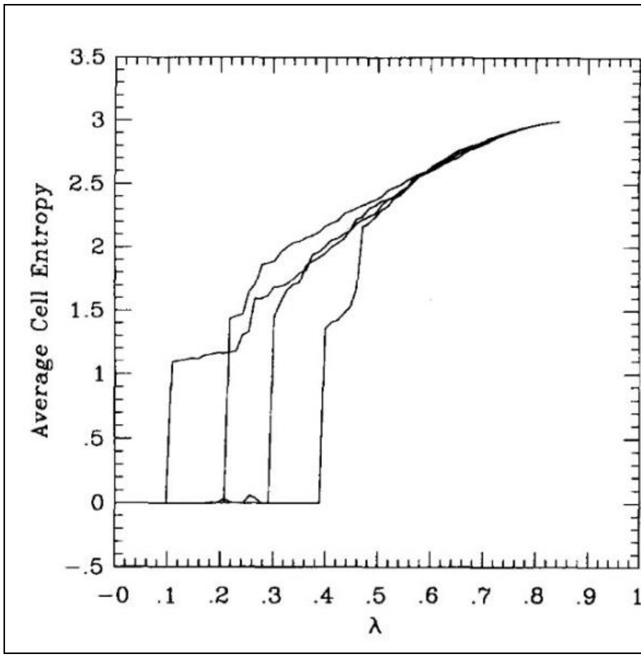
Many parameters may affect a system's stability and bifurcations in cellular automata such as *N*, the number of nodes, *K*, the in-degree of the nodes, and λ, the proportion of the logical rules for the nodes yielding a given state, e.g. 0 or 1 for the binary case. Langton (1990) explored this parameter and found bifurcations to chaos at a critical value, λ_c. Thus, low γ indicates fixed point or cyclic attractors, while higher γ indicates chaotic attractors. As with many systems there seems to be a typical route to chaos which progresses from fixed point to cyclic to chaotic to random. The region between the cyclic and random is sometimes called the ***'edge of chaos'***, but ***'chaos'*** works as well.

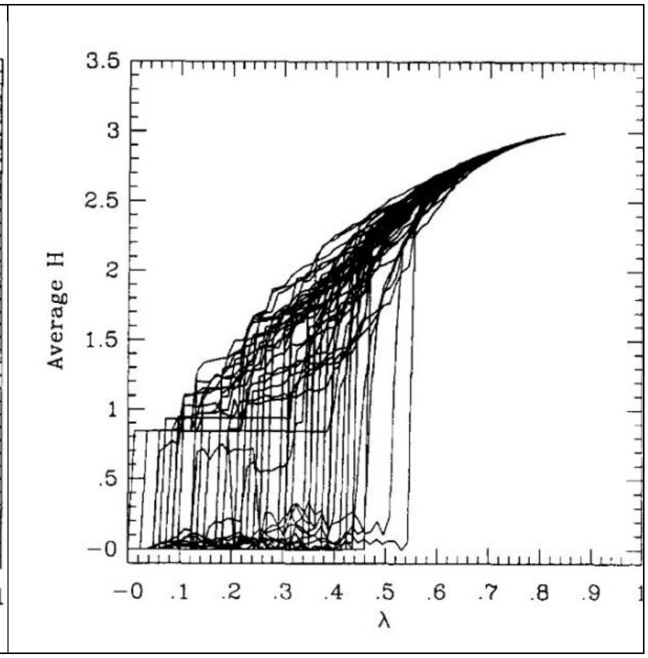


Spreading rates for a large ring network as a function of the proportion λ. It is modified from Packard (1988 and Mitchell *et al*. (1993). Several replications were run with new Monte Carlo seeds of the initial conditions, each with equal numbers of 0's and 1's, using genetic algorithmically generated changes in the rule tables to adjust λ. Ring lattices with r = 3, k = 2

Since this graph is based on the average of many simulations, we could ask if the gradual increase of γ might have involved some explosive bifurcations, quantum jumps along the way, an issue Langton (1990) explored with positive results using individual runs of λ.[17]

[17] This recalls the 'all-or-none' learning versus incremental learning debates of the 1950's and 1960's, and may provide some insight into resolving those debates. (Abraham, 1967; Estes, 1960: Hull (1943; and Voeks, 1954).





These figures of entropy as a function of λ show 4 (left) and 50 (right) sweeps of λ, each sweep for a given set of logical rules, showing both explosive and incremental bifurcations. What I want to suggest is that once the larger abrupt bifurcations occur, then as γ increases, the gradual increase in entropy is the result of a rapid sequence, a cascade, of small bifurcations. So we are not in an 'edge of chaos', but in a cascade of chaos-to-chaos bifurcations. The Ueda Explosion behaves the same way, with individual homoclinic trajectories being replaced by new heteroclinic connections between saddles, in a cascade. These are purely conjectural musings on my part; Ueda (1981) was somewhat mystified by them[18]. Thus the μ-bifurcation refers to the macro cascade of micro bifurcations as the control parameters pass through a relatively small regions, which is not an uncommon finding in networks (Grawer *et al*., 2015; Modes, 2016; Modes *et al*., 2014) and continuous (Alexander & Globus, 1996; Gregson, 1995) and hybrid systems (Eser *et al*., 2014) who refer to a similar process as 'deferred chaos' in neural self-organizing networks.

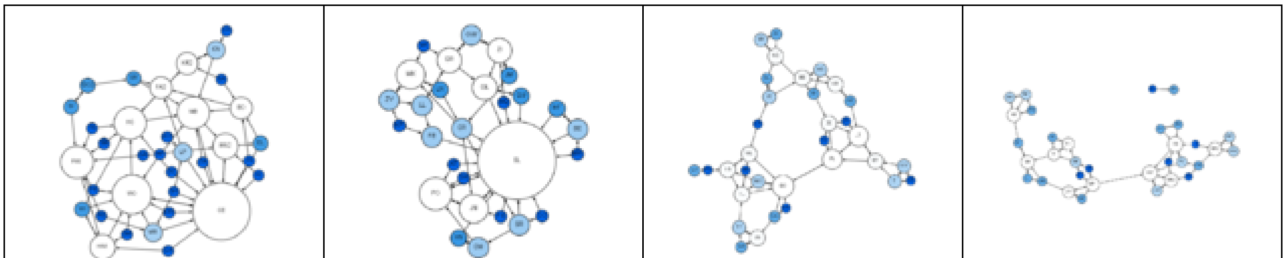
Cellular automata and their related homogenous networks are, like continuous dynamical system, often challenged to meet the demands real systems, and so contemporary network analysis has is evolving to meet those demands, using additional measures and a more intuitive graphics.

[18] "Duffing's equation has no statistical parameters and every solution is uniquely determined by the initial condition. The appearance of statistical properties in the physical phenomena in spite of the perfectly deterministic nature of the equation is caused by the existence of noise in the real systems as well as in the global structure of the solutions. A bundle of solutions representing the chaotically transitional process appears in certain domains of the system's parameters. The details of those stochastic regions have not been discussed as yet." (Ueda, 1980, pp. 422-434; 1992 and Kyoto University Research Information Repository, pp. 134-146.)

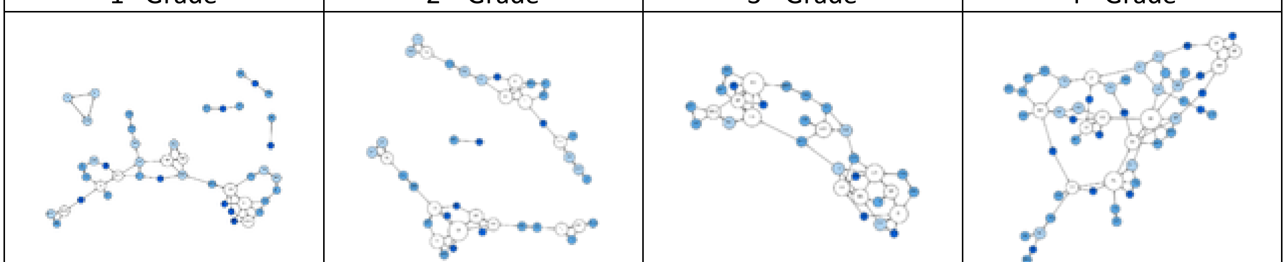
Historical Excursus

Euler's classic paper on his graphic solution to the Könisberg Bridges' problem suggests his solution as an example the more generally topological Leibniz's 'geometry of position' or *geometria situs* (Euler, 1735/1741). Euler's recasting the problem as vertices (nodes) and edges (connections) enable the cognitive leap to his theorem, and is often cited as the foundation of graph theory, although the development in that field languished pretty much until the developing of chemical bond theory in chemistry led Arthur Caley, 1874, the mathematician, to jump-started graph theory (Giaquinto, 2015; Weisberg, Needham, & Hendry, 2011).

The next major use of such graphic methods came within our field with Moreno's *Sociometry* (Grandjean, 2015; Moreno, 1934, 1951; Scott, 2013.) As one of three (the others were Heider and Lewin) Gestalt expatriates who fled Nazi Germany, it should not be too surprising that he used visual methods to portray phenomenological aspects of human relationships, nor that he used them to portray the evolution of such relationships.



1st Grade	2nd Grade	3rd Grade	4th Grade



5th Grade	6th Grade	7th Grade	8th Grade

These networks or 'sociograms' show the changes in friendship patterns in a class through its elementary grades and formed some of the basis for his explanation of a pandemic of runaways from a New York training school (Moreno, 1934)[19]. The method was used in many other social situations, and was also largely motivated by his group therapeutic methods, principally psychodrama which he introduced to the American Psychiatric Society in 1932.

Rapaport (1951) and Erdős and Rényi (1959-1968) were concerned with the evolution of network properties when links were added randomly among a set of nodes. Two nodes chosen randomly are connected with $0<p<1$. If a connection fails, then a new pair of nodes is selected at random; the process is iterated. One of their most important findings was that there was a bifurcation from small isolated clusters of networks to a giant component involving a large proportion of the nodes as the average number of links per node increases to one.

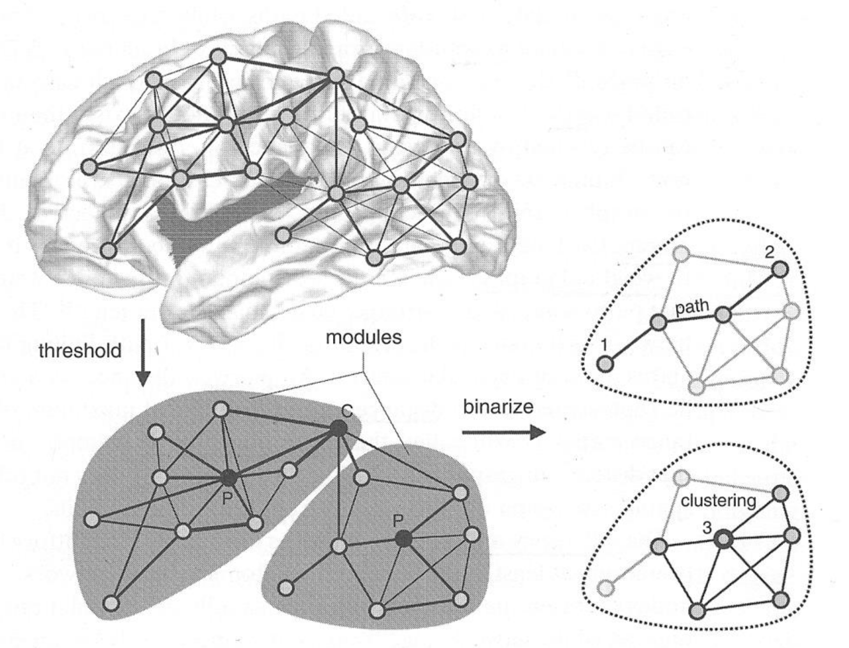
[19] Figures probably recreated by Grandjean (2015) using Gephi, retrieved 11 March 2016 from https://en.wikipedia.org/w/index.php?title=Jacob_L._Moreno&oldid=706753139

Unlike Erdős and Rényi whose interests were purely mathematical, Rapaport looked at the spread of information through a population and developed models of social interaction, and applied these to many areas including conflict resolution which mirrored his peace activism.

Network Analysis

Basic Measures

Many, if not most, networks are not homogeneous, nor are they stationary. Just as with continuous dynamics, some properties are more local in nature, and some more global. Consider the following figure which shows networks which could depict many different things from towns on adjacent islands set among brain coral, to nuclei set among various parts of the human brain (Sporns, 2011).

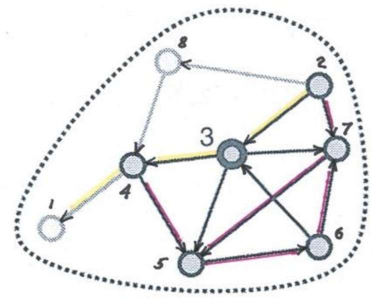


The clustering of nodes is made clearer by filtering out weaker connections. The modules each have a provincial hub, P, and there is a connector hub, C, between them. The lower module is shown at the right illustrating two of the major types of useful network statistics. The upper figure highlights the longest shortest path in the module, the 3 connections in the path between nodes 1 and 2. The lower illustrates measuring clustering. The **Clustering Coefficient**, CC, of node 3, is the fraction of connections among the nodes to which it is connected that to the total possible. The number of possible connections is 5!/(3!*2!) = 10; there are 5 such connection so CC = 5/10 or .5. The term 'binarize' means to ignore differences in importance (weighting) indicated by the thickness of the connections.

Clustering thus indicates how information is shared within local communities. Measures of clustering depend heavily on measures of degree, the number of connections per node. Global features are examined by the transmission between cultures or modules and over larger networks comprised of many modules. Measures of global features depend heavily upon path distances.

Most measures of network properties are obtained with the use of matrices, and I just give a hint of this and a sample of some of the measures. The **Adjacency Matrix** shows the connections between all pairs of nodes.   The connections may be weighted and directional, or not. Here is an example of a directed unweighted graph I made modifying the previous graph showing the **indegree** and **outdegree** for each of the 8 nodes, and their average for the network.

Adjacency Matrix for a Directed Unweighted Network



				receives								
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**		**avg**	
	1									**0**		**out**
s	**2**			**1**				**1**	**1**	**3**	**0.375**	**d**
e	**3**				**1**	**1**		**1**		**3**	**0.375**	**e**
n	**4**	**1**				**1**				**2**	**0.25**	**g**
d	**5**						**1**			**1**	**0.125**	**r**
s	**6**			**1**				**1**		**2**	**0.25**	**e**
	7					**1**				**1**	**0.125**	**e**
	8				**1**					**1**	**0.125**	
		1	**0**	**2**	**2**	**3**	**1**	**3**	**1**	**13**		
		0.125	**0**	**0.25**	**0.25**	**0.375**	**0.125**	**0.375**	**0.125**		**0.203**	
					average indegree							
					indegree							

A high degree of clustering makes for many connected triangles, another measure of clustering sometimes aka 'transitivity', which may indicate a sharing of information, and an increase in redundancy or coherence. "Highly modular graphs often consist of densely clustered communities . . . Despite their partial redundancy, each measure of local connectivity also provides some unique information about the way individual nodes are locally imbedded … and about their community structure…" Densely connected clusters are "likely to constitute a functionally coherent … system." Nodes in different clusters may not share much information and "remain functionally segregated from each other." (Quotes and précis from Sporns' pp. 12-14.)

Hubs can be well connected, integrative, and can compensate for network damage. In networks with large degree variance, they have high degree, provide between-cluster connectivity, and great centrality. They can be measured by the **Participation Coefficient** (the ratio of the degree of node within a cluster to the average degree of all nodes). When high, hubs facilitate inter-cluster communication and integration; connector hubs have high participation coefficients. When low, they indicate provincial hubs that can promote communication within the cluster.

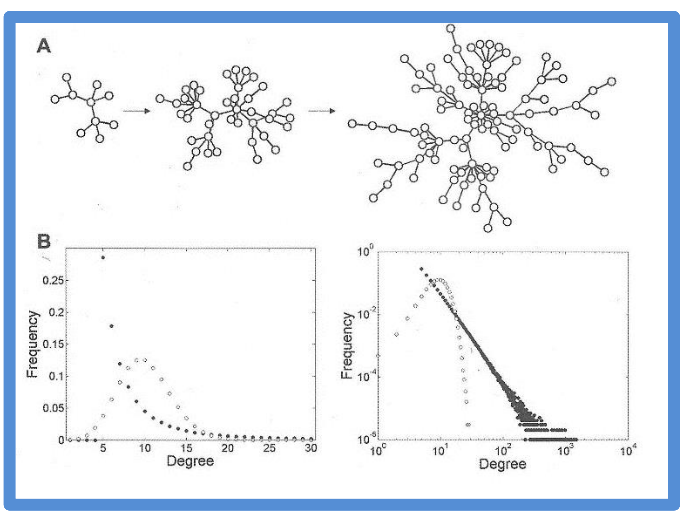
Path length becomes important, and thus the **Distance Matrix** becomes paramount. Distance for a weighted network means not only the number of links in the shortest path between two nodes, but, if weighted, the length of each link is the reciprocal of its weight. That is, more important links make the path 'faster' or 'more efficient' which implies it can be considered a shorter path, more useful for network integration. Globally, path length is the average of the distance matrix. Integration can also be aided by multiple paths between nodes; redundancy.

Centrality measures control of network communication that depends on short paths. **Closeness Centrality** of a node is the reciprocal of the average path length between that node and all others. **Betweenness Centrality** of an individual node is defined by the fraction of all shortest paths that pass through it. **Eigenvector Centrality** "is based on the principal eigenvector of the [network's] adjacency matrix" which can include longer paths." (Sporns, 2011, pp. 15-16.)

Types of Networks

Some of the most studied types of networks are:

1. Random: Connected by equal probabilities; homogeneous degree distribution (Solomon & Rapaport, 1951; Erdös & Rényi, 1959).
2. Lattice: Homogeneous structure and connection rules; connections are with nearby neighbors in 1D (ring) and 2D or 3D grid lattices. Higher clustering; longer path lengths. (Langdon, 1990; Kauffman, 1995; Packard, 1998; Mitchell *et al*.; 1993; etc SFI; Abraham, 2015b; Watts & Strogatz, 1998).
3. Small World: Connections in empirical networks are also made stochastically which yield high clustering and short path lengths. (Pool & Cochen, 1978; Milgram, 1967; Easley & Kleinberg, 2007; Watts & Strogatz, 1998) Small world index: Ratio of normalized Clustering coefficient [large] to normalized path length [small], if >1 then small world topology. (Humphries & Gurney, 2009; Humphries, Gurney, & Prescott, 2006). Test results by omparing to random network with same degree distribution (Wiki entry, see note this slide; includes ω def).
4. Scale Free: [Stochastic] preferential attachment evolution; probability of connection is random or may depend on some property of the nodes, such as their degree. The frequency is a logarithmic, not Poisson (includes normal) function of degree. (Barabási & Albert, 1999.)



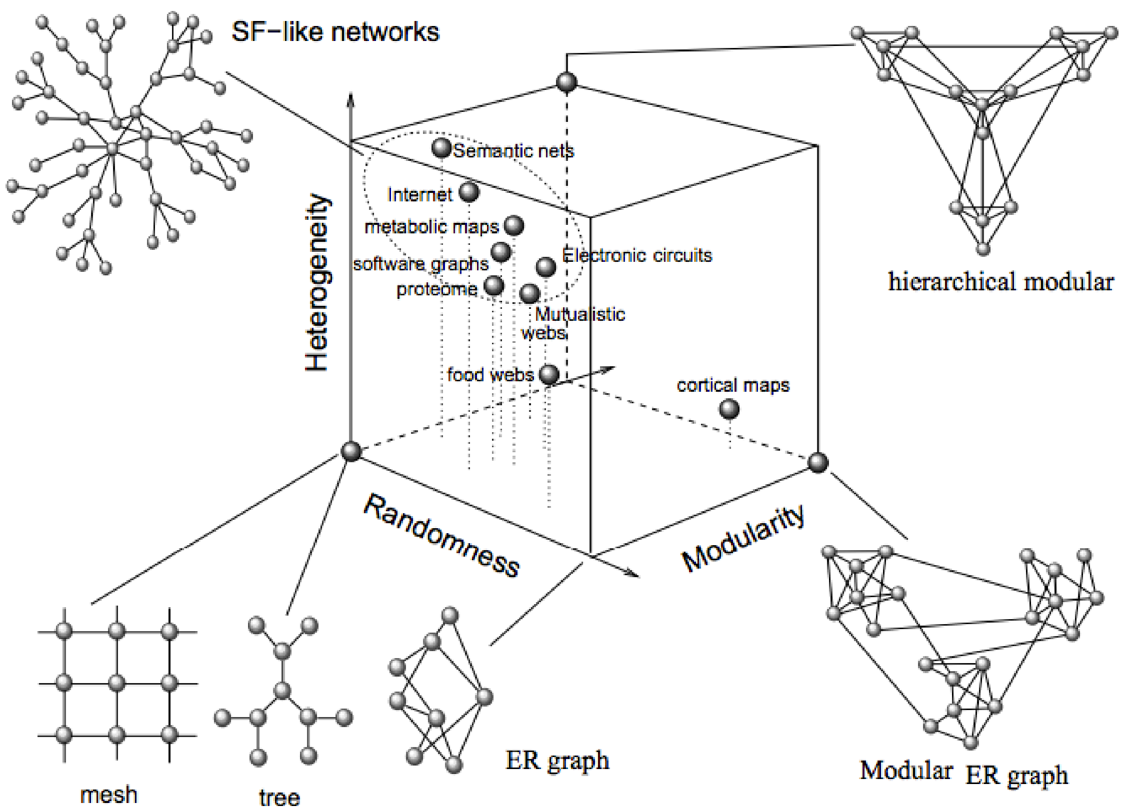
A: growth of a preferential attachment network.

B: linear and log log functions of degree for scale-free (black dots) vs random (open dots). (Sporns' figure 2.4, p. 21.)

'Scale-free' refers to the rate of change of the log log function being constant throughout the whole range of degree.

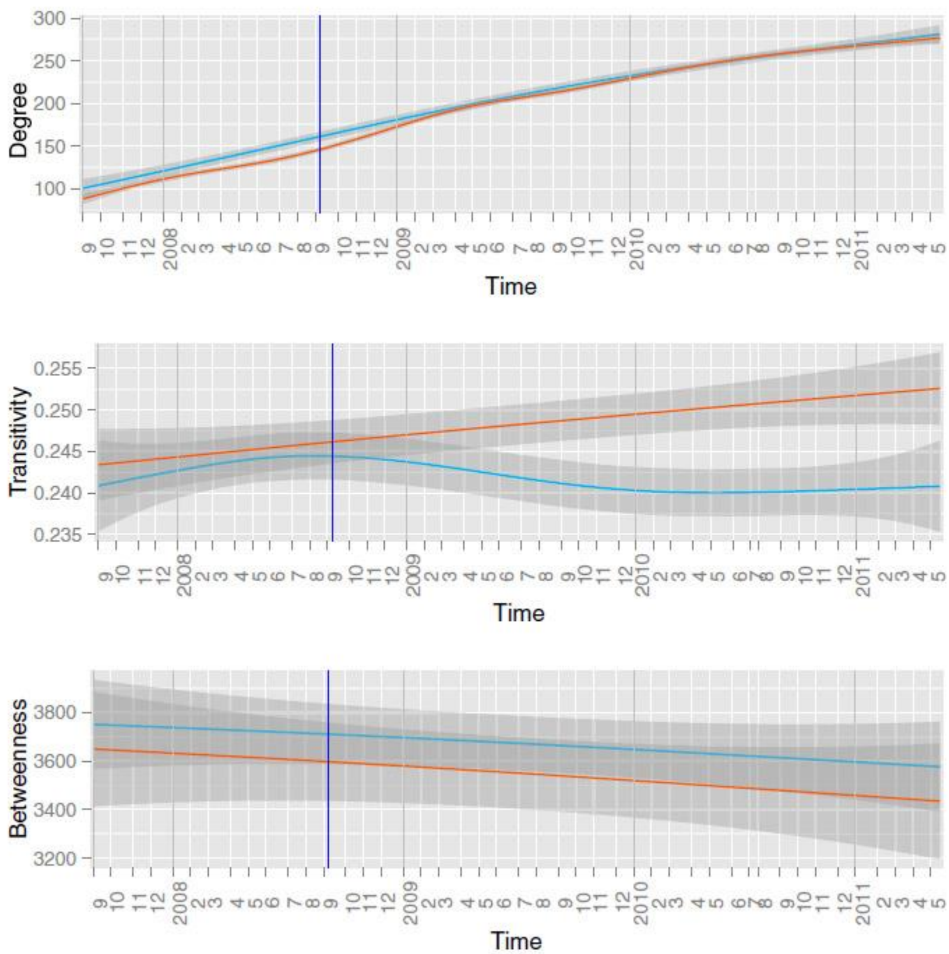
There are many variations on networks that possess these scale-free properties, which include pruning as well as adding nodes and connection s. In addition to showing this property of a power law, such networks can show bifurcations, chaos, self-organization, and the μ-bifurcation process (see Barabási,2002 , esp. Chapter 6.)

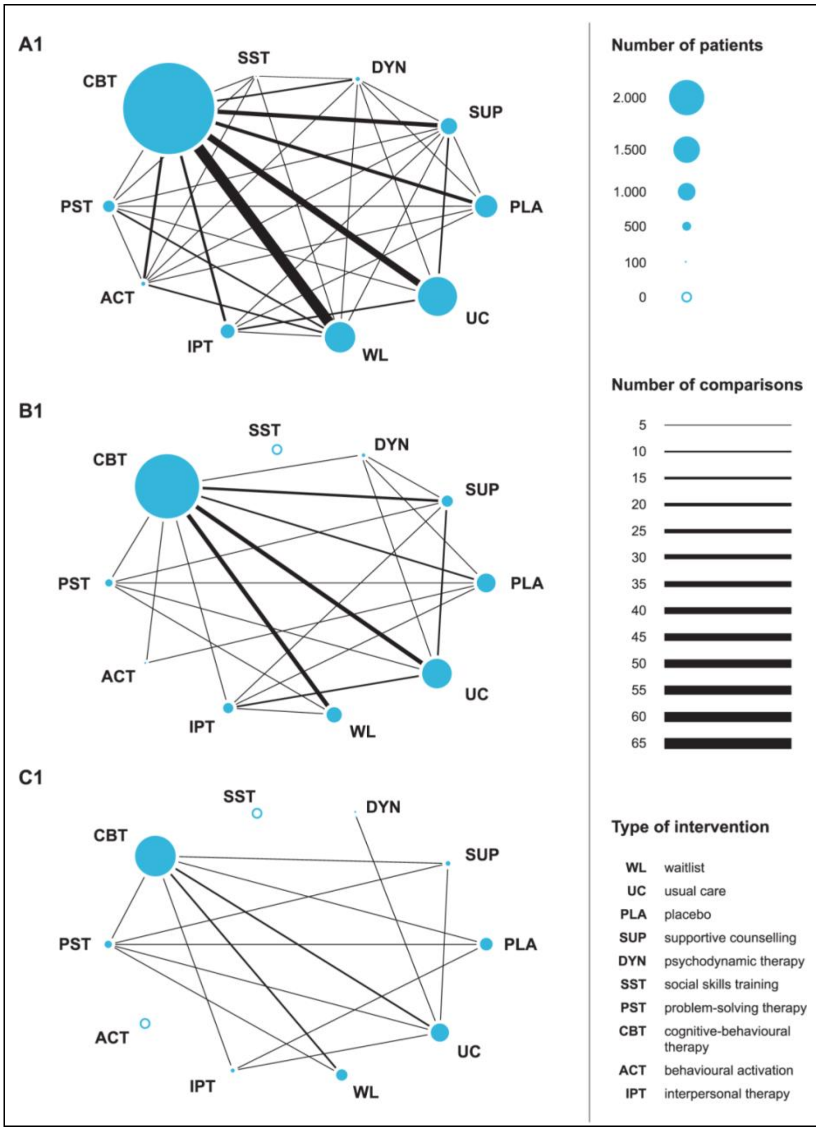
A more comprehensive view of varieties of networks has been shown by in a 3D space based on network heterogeneity (degree distribution), randomness if adding and pruning of links and nodes, and modularity. (Solé & Valverde, 2004, fig. 3.) (ER for Erdös & Rényi; SF for Scale-Free.)



Some Examples of Network Analysis

Phan & Airoldi (2015) examined Facebook interactions among thousands of students in various US universities and colleges over a number of years, before and after a catastrophic Midwestern USA storm. Some of the schools suffered the effects of the storm; the control schools did not. The schools were equated for various parameters, but not for the pre-storm equivalence on the principal dependent variables, three of the most prominent network variables, average degree, transitivity (average cluster coefficient), and betweenness centrality, notice the pre-storm disparity for betweenness centrality. The study illustrates the use of big data and the selection of critical measures of the major types of measures from among the multitude possible. You might call this a negative pilot result, and then decide to move on to another problem to study, or to probe some modules for local results or. In so doing, one might encounter particular factors of group dynamics. Such a study would illustrate a move toward a more meta-level of analysis. In the figure below, red lines are the schools affected by the storm; blue the control schools.

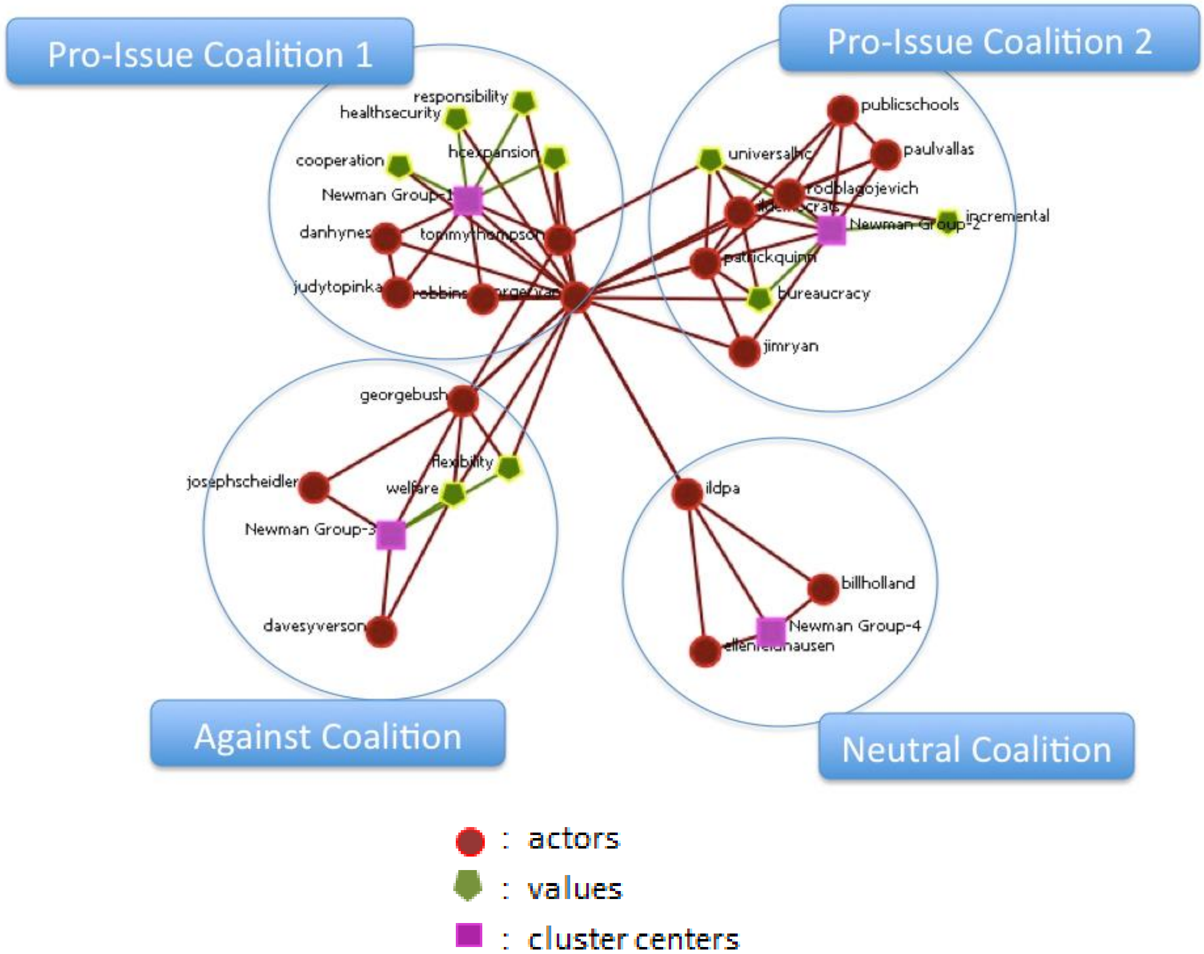




Barth et al. (2013) surveyed 198 studies of the efficacy of psychotherapeutic interventions for depression using "network meta-analysis, a novel methodological approach that integrates direct and indirect evidence from randomised controlled studies." All therapies were "superior to a waitlist control condition" and "Interpersonal therapy was significantly more effective than supportive therapy." "Stepwise restriction of analyses by sample size showed robust effects for cognitive-behavioral therapy, interpersonal therapy, and problem-solving therapy compared to waitlist. The meta-analytical aspect not from portraying the dynamics of therapeutic methods per se, but in portraying the nature of the distribution of methods between studies of their differences.

One of the best examples of meta-analyses I have come across were done by Soohee Kim (Kim & Morçöl, 2015) for her doctoral dissertation; she was studying the evolution of coalitions of individuals and values of civic groups developing governance projects, in the example given here, in an Illinois childrens' health insurance program. She used tools developed by Carley at Carnegie Mellon (Carley, 1997; Carley *et al*., 2009). The figure here shows the individuals, the social values they considered, and the hubs for each coalition as separately coded nodes in the same networks. This figure represented the third of four stages of evolution of the coalitions.[20]

[20] Besides her presentation at a governance conference, I encountered this study at a joint conference by Auburn University's School of Public Health and the Winter Chaos Conference, 2015, presented by Morçöl



While in this example there are too few nodes of each variety to expect computation of network statistics to add to what is reasonably apparent from the figure, I think it can be surmised that this figure offers a canonical representation of principal components of coalitions, values, and actors and their functional relationships. This is not a mere metaphor when one considers that matrix computations are used in such an analysis, and would continue to play a role in further partitions of the matrices to probe local impact of these factors. Note that network statistics could be coded in the size, shape, and color of nodes, and thickness and color of connections. Canonical methods, pioneered by Pearson (1901) and used in many areas of linear and nonlinear canonical multivariate analysis are variously called principal component analysis, eigenvalue decomposition, singular value decomposition, and factor analysis. I once showed that transactions among brain loci in an EEG study of recovery of function in the brain revealed a trajectory in their factor space from a step-wise discriminant analysis, a linear application (Abraham, 1997; Abraham *et al*., 1973

EPILOG: SUMMARY & CONJECTURES

To summarize our trajectory so far. We have introduced some basic concepts of two important types of complexity, dynamical systems and networks and their basic graphic representations and measures. We have made the contention that chaos characterizes the majority of psychological and biological systems of interest to us. The conditions under which they evolve, including stability-instability, bifurcation, μ-bifurcation cascades, self-organization/emergence, the control of chaos, and the suggestion that most bifurcations are chaos-to-chaos, have been emphasized. We have also emphasized that when using non-traditional statistics, that *monte carlo* methods of scrambling data while maintaining some of its original parameters, is often the principal way of showing your data is not random.

A note on chaos as good or bad. Chaos is not necessarily good nor bad, but we can make remarks on the conditions under which health, happiness, creativity, and adaptability can best flourish. Chaos can be beneficial when mid-dimensional, and the object of therapy may depend on the control of chaos, that is keeping it mid-dimensional where creativity and change can best be accomplished (Alford & Abraham, 2011; Guilford, 1953; Krippner *et al*., 2014; Richards, 2000, 2001, 2007; Schuldberg, 2007). The abnormal states seem to be too low dimensional, e.g., catatonia (nearly fixed point attractor) or obsessive, bipolar (some aspects being nearly cyclic). it can be admitted that circadian driven biological and psychological rhythms can be fairly low dimensional, although I think that research will show that more variability will be shown to be better than a rigid daily schedule. Even the planetary-solar rhythms are chaotic, though very low dimensional. They can also be too high dimension.

Science as a creative place to play. Perhaps the point I would like to stress most is that science is not a fixed strategy for obtaining the "Truth", rather it is an occupation, preoccupation in which our curiosity and creativity leads us to provisional ideas and results, all of which are under constant revision. For example, the fact that most problems we encounter could be tackled with either dynamical or network tools or both, as my comparison of them showed, but each tool and measure gives a little more insight, another perspective of the processes we investigate, or as in therapy, try to navigate. John Dewey (1960, Bernstein, 1985) emphasized a similar point of view on philosophy, science, democracy, education, and Bridgman emphasized them in his second book on operationalism and the vibrancy of being a scientist (Bridgman, 1936; see also Chang, 2009).

Finally, in view of the fact that most instantiations of our systems are closed sets of equations or of nodes and links, what is all the fuss about in bringing complexity to almost a state of religion (see Robertson, 1996), the passion and wonder that drove our commitment to it in the first place. In moving from the old paradigm, to the new, we are always aware of the importance of contextual elements, that systems in isolation, while complex, are never complete. Dynamics makes us aware of the totality of the effort in which we are involved. Ralph and I have felt that dynamics, as itself dynamical self-organizational, represents a μ-bifurcational process to yet new levels of personal and cultural consciousness, of self-consciousness. On a personal note, I amend such an observation with my view that complexity/dynamics provides me with additional conceptual tools, whether formal or metaphorical, for a fluid experimental approach not only to science, but to life, personal and communal, and to contributing to socio-psychological transformations. I suspect many of the contributions to this volume will contribute to my understanding of these processes.[21]

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[21] Related ideas include Abraham (2007) and *The Dynamics of Culture*; *Media Ecology, Globalization, & Emancipation: Beyond the Carnivalesque*; *Sophia*, and *Topoi and Transformation*, can be found at: http://www.blueberry-brain.org/chaosophy/index.html

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