

A BEGINNER’S GUIDE TO THE NATURE AND POTENTIALITIES OF DYNAMICAL AND NETWORK THEORY

PART I: A VERY VERY BRIEF VISUAL INTRODUCTION TO THE THEORY OF DYNAMICAL SYSTEMS

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“Concerning things which mortals cannot perceive, only the gods know, but for we mortals, only conjecture from sensory evidence [tekmairesthai] is possible.”⁵
(Alcmaeon, circa 500 BC.)

“... a substantial part of the theory of organization will be concerned with properties that are not intrinsic to the thing but are relational between observer and thing”⁶
(Ashby, 1962).

“General systems theory has taken its impetus from the excitement of discovering larger and larger contexts, on the one hand, and a kind of microprobing into fine detail within a system, on the other. Both of these activities are intrinsic to anthropology to the extent that field work in living societies has been the basic disciplinary method. It is no

¹ The author is the beginner, who invites others to share this introductory exploration. I minimize citations to those that are my principal sources and serve as a gateway to the vast literature on these subjects.

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³ Except where other authors or publishers have previously copyrighted cited material or where internet sites have presented non-copyrighted material cited herein.

⁴ This essay is a summary of some basic principles of dynamics that I mostly learned and redacted from my brother, Ralph (Abraham & Shaw, 1982-88; 1987; 1992, 2005), updated with some of my own selection of additional materials gleaned from various sources, especially from the worldwide web where great graphics and interactive applets abound for those wishing to venture further. I have asked his supervision for this latest redactive effort in order to minimize any errors or distortions.

⁵ Motto at beginning of Alcmaeon’s *On Nature*. The statement is my merge of commentary by Huffman (2013) and Manetti (1993) based on Diels & Kranz (1951-4, 1972/2001). Alcmaeon’s theories on health were dynamic (balance of multiple binary oppositions) and he was the first of the early Greek Enlightenment philosophers to specify the brain as the locus of understanding.

⁶ ‘Thing’ is a subset [aka attractor] in the ‘product’ [aka state space] representing a set of interacting variables. This quote hints not only of uncertainty in the observer, but also of the arbitrary nature of defining variables. It hints of Korzybsky’s map-territory distinction (1931/1933), and the unity of science movement as well (Neurath *et al.*, 1938). These quotes suggest both the potentialities and limitations of systems theory and math-science.

revelation to any field- experienced anthropologist that everything is related to everything else, or that whether the entire sociocultural setting can be studied in detail or not, it has to be known in general outline.” Mead, 1973.

“Viewed against the background of the movement of thought from Kant through Hegel to Marx, . . . conceptual substitution . . . [of] thought-figures are retained in structural analogies. . . The system-environment relationship is thought out altogether in accord with the model of a world constituted by transcendental consciousness. . . The systems theorist finds a multiplicity of system-environment relationships in his object domain. . .

“Systems theory lets cognitive acts, even its own, meld into the system’s achievements of mastering complexity and thus takes away from knowledge any moment of unconditionality.” (Habermas, 1995).

DYNAMICAL SYSTEMS

A *dynamical system* is, loosely speaking, a set of variables interacting over time. The changes in these variables, their time series, can exhibit various patterns of behavior. If there are reasonable regularities and smoothness in these patterns, they can be represented by graphic methods and mathematical notation. The equations describing the changes over time—usually ordinary differential equations (ODE’s) for the continuous time case, or difference equations for the discrete time case— may contain nonlinear elements. These empower the ability of such nonlinear models to describe not only point and cyclic activity asymptotically but also are necessary (but not sufficient) conditions for the production of ‘deterministic chaotic’ activity as well—often known as *chaos, chaotic attractors or strange attractors*. These nonlinear elements also are necessary for the production of dramatic shifts in the temporal patterns generated by the systems—*bifurcations*.

2. THE VECTORFIELD

Dynamics describes change over time. For every point in some *state space*, a vector describes the tendency to change at that point, specifying the direction and the rate of change. The *vectorfield* is the collection of these vectors. The state space may use physical variables, such as meters or volts, or that of some non-physical variables, such as sweetness or population density. Here are some examples. On curved spaces, the vectors are tangents to the surface or curve. Graphical geometry or the vector notation of vector calculus may represent the *vectorfield* (Figure 1).

3. TRAJECTORIES

A system generates time series of the variables of the system. These can be represented in the usual fashion, as a function of time, or collapsed over time, in which case the *trajectories* can be visualized in the state space with time and vector coding from a variety of methods (see Mathworld demonstrations in the Additional Resources Box above).

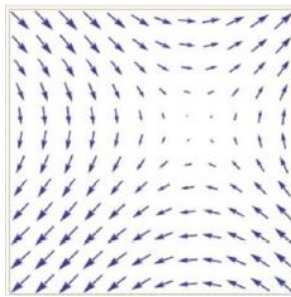


Figure 1a. Two-dimensional Vectorfield in a planar state space.⁷

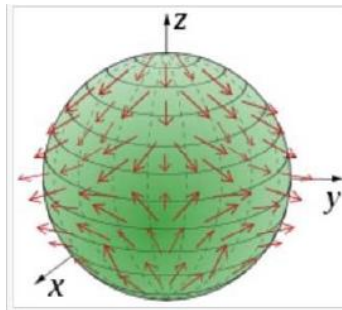


Figure 1b. 2D Vectorfield for a spherical-surface state space.⁶ (Tangential to surface.)

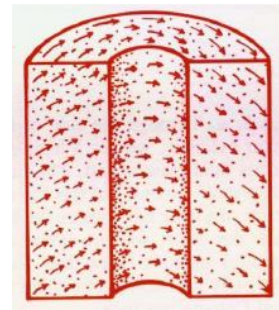


Figure 1c. 3D Velocity Vectorfield. For the Couette Stirrer.⁸

Additional Resources:

<http://www.falstad.com/vector/> Terrific interactive applets of 2D and 3D motion.

<http://mathworld.wolfram.com/VectorField.html> Article plus links to 4 demonstrations; needs

Mathematica Player 7 (easy download) for the interactive demonstration projects.

<http://tutorial.math.lamar.edu/Classes/CalcIII/VectorFields.aspx> Paul Dawkins' class notes

You can think of a system being at a particular state, a point in state space, where it obeys a particular vector, which moves it to its next state, where another vector moves it along, iterating for as many steps as you wish, a process called integration (of the ODE's) in calculus. Here are some *trajectories* superimposed on their *vectorfields* (Figure 2).⁹

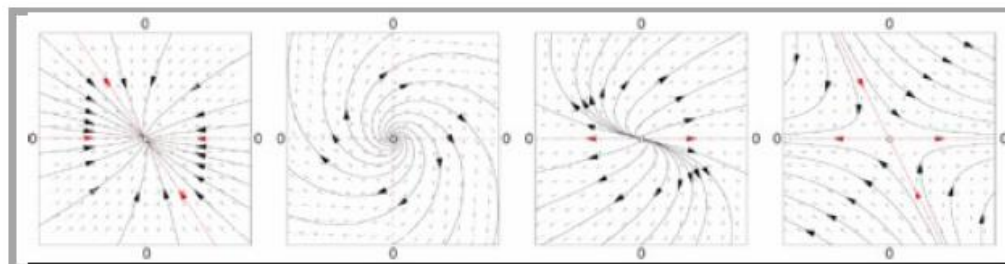


Figure 2. Two-Dimensional *Portraits*. From the left: *trajectories* converge to a *point attractor* with negligible spiraling. The rate of travel toward the point is expressed in a real component of the vectors, and the rate of spiraling is in the imaginary component. Next to it is a similar convergence to a point, with generous spiraling. Next is a *point repeller* with *trajectories* showing a fair amount of spiraling? On the far right is a *saddle point*, with some *trajectories* approaching and some departing. The inset forms a *separatrix* between two *basins* of attraction.

One may code the velocity of *trajectories* using codes superimposed on them, such as the use of tick marks spaced at equal time intervals or by color.

⁷ http://en.wikipedia.org/wiki/Vector_field As modified on 16 November 2009 at 01:15.

⁸ Abraham & Shaw, 1988, p. 30 & 1992, p. 472; 2005, Fig. 16.3.2.b.)

⁹ http://en.wikipedia.org/wiki/Dynamical_systems Retrieved 21 November 2009.

Question: Can you draw a trajectory and indicate with tick marks where it is moving faster and where it is moving slower?

Since *trajectories* are evolving in time, they constitute *time series*, and one may plotted them as such, that is, as a function of time as well as in the *state space* representation. The same holds true for the *vectorfield*, though that is not as commonly done.

Question: Can you draw a trajectory both in a state space and as a time series for a *periodic attractor*?

4. BASIC ATTRACTORS

Trajectories may tend to some basic patterns, which are called *attractors*. Some of the most basic types are fixed point (Figure 2, left two panels), periodic (cyclic), toroidal (with irrational winding ratio, otherwise they would be periodic!), and chaotic, those being from nearly cyclic (e. g., the Rössler Attractor) to nearly random.

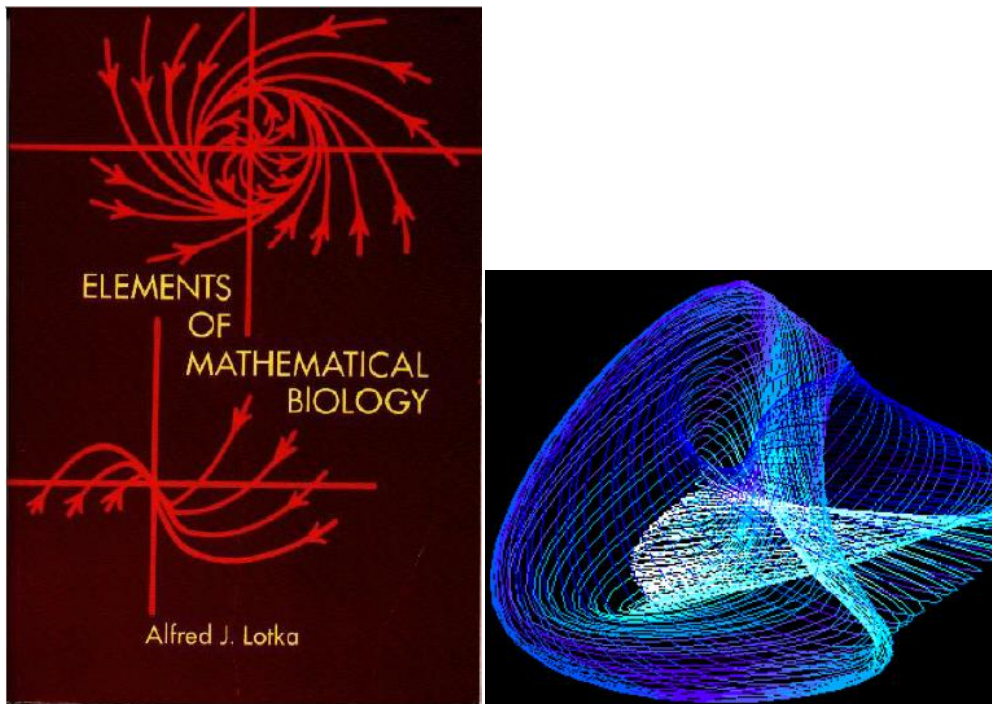


Figure 3. Attractors for Prey-Predator Models (axes: density of each species). Left: Cyclic Attractor with point Repellor (top), and Fixed-Point Attractor (bottom)¹⁰. Below: Chaotic Attractor for Prey-Predator with Two Habitats.¹¹

¹⁰ Lotka, 1956, cover.

¹¹ Schwinning at Schaffer's web site.

5. BASIC FEATURES REVEALED IN THE PORTRAIT: ATTRACTORS, REPELLORS, SADDLES, INSETS, OUTSETS, BASINS, SEPARATRICES

The dissipative Euler Buckling column can illustrate many of these features. Imagine a plastic column or membrane, compressed on the ends or edges, which is then set to vibrating in a plane and comes to rest to one side or the other of the center of the axis or plane of vibration. A typical state space, as for many similar phenomena such as the swing of the pendulum, is comprised of the position versus the first derivative of the position, the velocity. Such a portrait is shown in Figure 4.

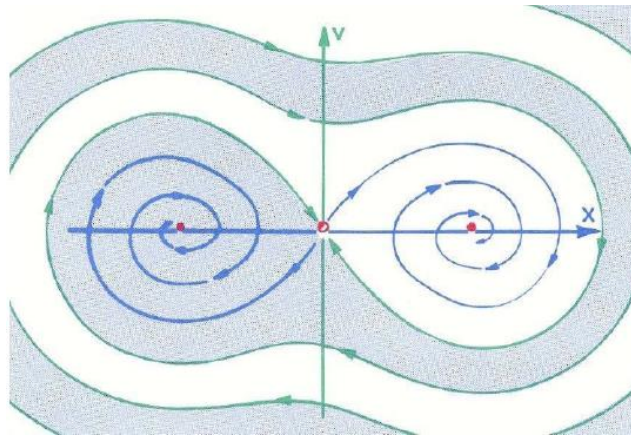


Figure 4. The Dissipative Euler Buckling Column¹². There are two focal point attractors, each with its own basin of attraction, in which all trajectories spiral in to the fixed point attractor. There is a saddle point in between. The trajectories approaching the saddle point each form a separatrix, a boundary between the basins. Note the similarity of the central part of this portrait to that of panel 4 of Figure 2. This model has been used in civil engineering (bridges, buildings, such as the collapse of the twin towers, NYC, 1991, aeronautics, and presumably economical and various psychological and social applications. Its first application in psychology was by Guastello (1985). I see it in action on the ski slopes. No not the skis, but rather my body. Lactic acid is the control parameter, forcing the trajectories toward focal point attractors to one side or the other of the ski trail. **Question:** Can you imagine a psychological or social phenomenon that displays similar behavior?

Historical Note. On September 14th, 1953, N. J. Hoff delivered the Wilbur Wright Lecture on *Buckling* and stability to the Royal Aeronautical Society and the Institute of the Aeronautical Sciences (The Wright brothers received the first Gold Medal of the RAS in 1909), which included “an analysis of the dynamics of the motion of a column during the buckling process in a normal column-test, beginning with the equations of motion of a perfectly elastic column.” and a dynamic theory for short columns. It also included a discussion of “the relation between the static and dynamic criteria of stability. Phase diagrams were then shown . . .” (*Flight*, 1953.).

¹² Figure 2.2.6, p. 70, from Abraham & Shaw, 1982.

Other features were shown above in Section 3 Trajectories and Section 4 Basic Attractors. The portrait here is to show that many features can be revealed in a portrait, which technically includes or represents all possible trajectories of the system.

6. RESPONSE DIAGRAMS

We have established that a given dynamical model or system may provide different patterns, portraits, system behaviors, from the interaction of its variables over time, and that the differences in those patterns, while produced from the same mathematical equations, are due to different values of the system's parameters. This property of being able to summarize different system behaviors with one set of equations is one of the great offerings of dynamics— parsimony. Besides depicting the behavior of the system as portraits in state space, we can also map the territory occupied by each type of portrait in parameter space. There are various terms one might use for these maps, such as the *response diagram*, the *bifurcation diagram*, or the *bifurcation portrait*. There are many ways to do this. Here is one of the simplest for one of the most used and abused objects in dynamics, the cusp catastrophe (Figure 5).

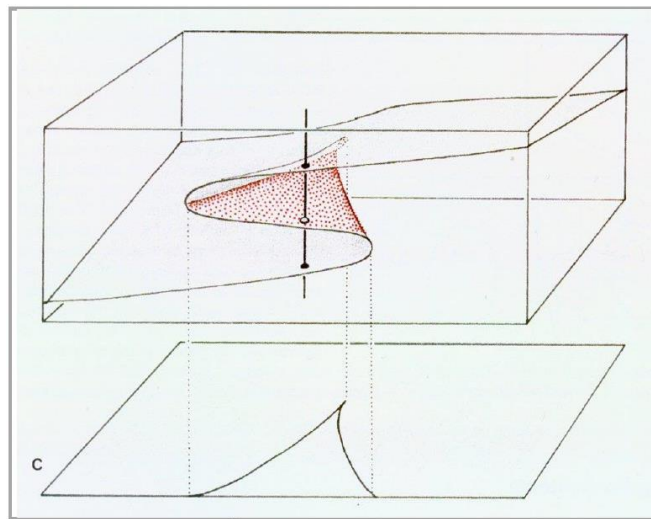


Figure 5. The Cusp Catastrophe¹³. The state space is one-dimensional, a single variable. The parameter space is two-dimensional. One usually represents them in their joint space as in the upper part of the figure. This is the 3D *Basin-Boundary Portrait* of the system. The vertical line shows the state space for one point in the parameter space. Its portraits consist of the all the trajectories that move toward each of the point attractors (solid dots) depending on which side of the repellor (open dot) is their initial state. The folded blanket part of the surface shows the loci of the attractors (solid dots) for each point in the parameter space (upper and lower layers in the folded part, and shows the loci of the repellors (middle red-dotted layer). Outside the area of the fold, there is only one point attractor, and no repellor. The projection to the bottom of the figure shows the response diagram in the 2D parameter space, with the triangular area being the area where the

¹³ Figure from Abraham & Shaw, 1987, p. 583, with kind permission.

system has two attractors and one repeller, and the space outside showing the area where there is only one attractor. **Question:** Can you see how these features generate the hysteresis effect from systematically changing the parameters? **Hint:** As one decreases the parameter from right to left, when entering the area of the fold, in which basin of attraction is it? Likewise, when increasing the control parameter from left to right, in which basin is it after the bifurcation from one to two point attractors?

The familiar figure for the logistic equation is another example. **Question:** Can you draw, either by calculating and sketching by hand, by using a hand calculator, or by using a spreadsheet on a computer, the response diagram for the logistic equation (the equation is $x_{(n+1)} = \lambda x_n(1-x_n)$, usually explored with real numbers such that $0 < x_0 < 1$, $0 < \lambda < 4$)?

7. BIFURCATIONS

A bifurcation occurs when there is a significant change in the portrait as some parameter changes and passes through a threshold called the *Bifurcation Point*. For the cusp shown above, the loci of the bifurcation points are given by the line outlining the tent-like area in the parameter space (the lower part of Figure 5). As the parameters move into or out of this area, a fold bifurcation occurs. The cusp thus is sometimes called a double fold. The diagram thus can also be called a *Bifurcation Diagram* or map. If you did the exercise for the logistic map, you probably discovered that the response diagram reveals a sequence of bifurcations, the bifurcation sequence, for that equation. **Exercise:** try running the sequence going from $\lambda = -2$ to 4. You can check your result at <http://www.hevanet.com/bradc/MiscMathStuff.htm> where there also is a very complex and colorful map for when λ is complex—Figure 6). The cusp and the logistic are iconic fetishes of the dynamics community, from which we learn much, but for which the applications tend to be trivial in practice.

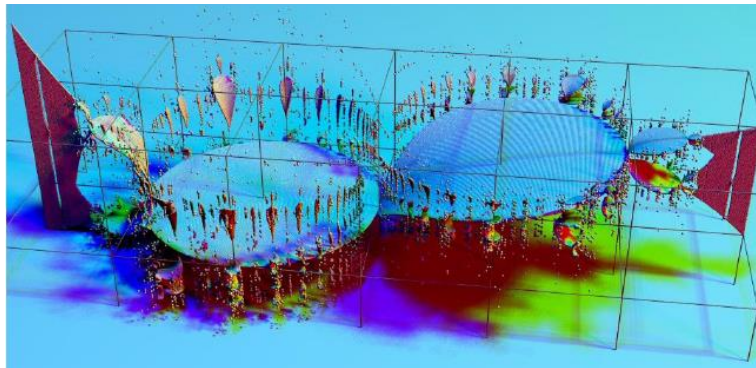


Figure 6. Bifurcation Diagram for the Logistic Equation with Complex and negative λ ¹⁴.

This is how fractals are plotted in a complex 3D space. **Exercise:** Can you find the traditional logistic bifurcation diagram within this diagram? **Hint:** Check out the vertical real plane where the imaginary component of $\lambda = 0$. (the vertical x -axis for the variable x , left-right axis is for the real component of λ , and the front to back axis is for the imaginary component of λ).

¹⁴ Chalfan, 2002.

8. CHARACTERISTIC PROPERTIES IN PORTRAITS

It is common to establish some basic properties of limit sets and their insets and outsets, such as the rate of approach and departure of trajectories from points and cycles¹⁵.

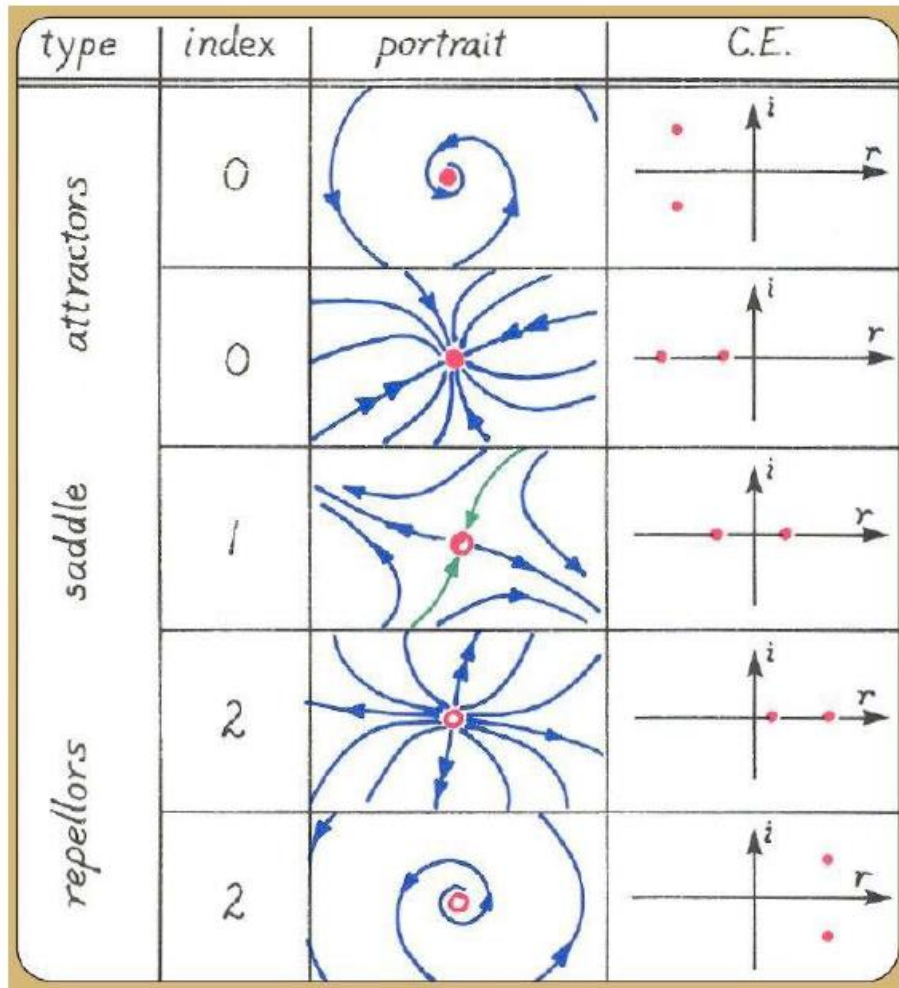


Figure 7. *Characteristic Exponents (aka eigenvalues) and Indices for Typical Limit Points in Two-dimensional State Spaces.*¹⁶

The characteristic exponent (CE) of a critical point is a complex number whose real component represents the rate of approach and departure of the trajectories, while the imaginary component represents the rate of rotation of the trajectories. These points are

¹⁵ We give only a few here to establish the basic ideas, but the subject should be explored more fully for limit points and cycles in one, two, and three dimensions in Abraham & Shaw, Parts 2 & 3, which should also be consulted for generic properties relating to stability and Peixoto's theorem relevant to the next section.

¹⁶ "The geometric representation of complex numbers [*Argand diagram*] was championed in 1806 by Jean-Robert Argand (1768-1822), a French bookkeeper and amateur mathematician." From Sprott, 2003, p. 75.

hyperbolic, that is, the real component is not equal to 0. The index is the dimension of the outset of a critical point.

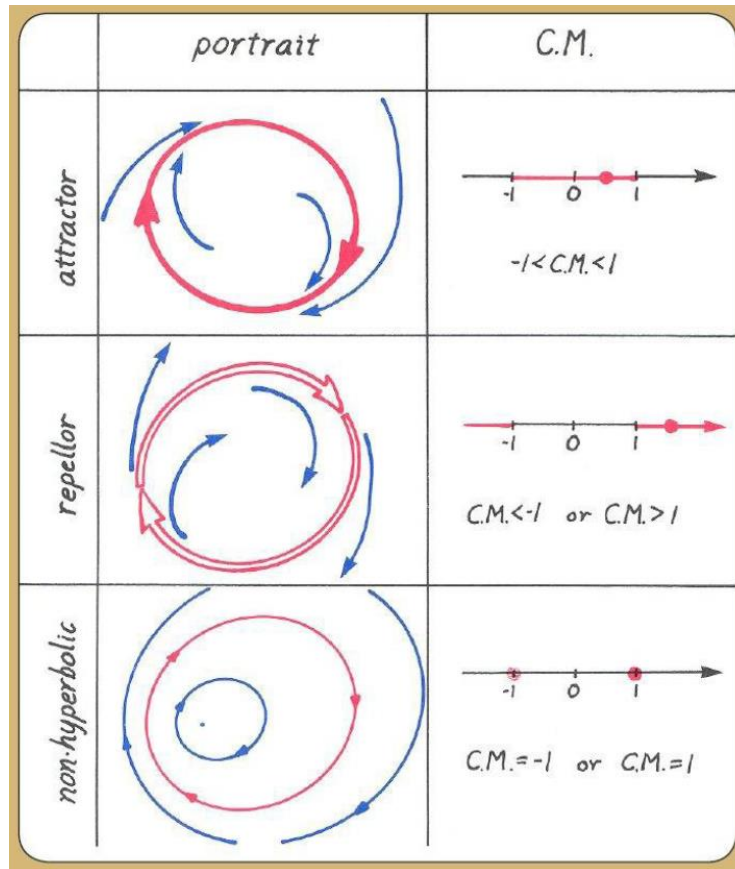


Figure 8. *Characteristic Multipliers* (aka *eigenvalues* or *Floquet multipliers*) for *Limit Cycles* in *Two-dimensional State Spaces*. Note the unstable center (non-hyperbolic case) in the bottom panel. The CM is never 0. Complex CMs exist for the three-dimensional case; we omit them here and again urge consulting the works of Abraham & Shaw; also Sprott. For an interesting application of complex CMs in 3D, see Duchesne (1993).¹⁷

9. STABILITY

Abraham & Shaw describe a stable or unstable system by how it is affected by a perturbation that is as an addition to the *Vectorfield*. For example, the center just mentioned, is unstable, but their figure (Figure 9) shows how easily a perturbation can change the portrait into a point attractor or repellor.

The generic properties that describe a stable system (Peixoto's theorem) summarize a necessary and sufficient set of conditions for stability. However, these do not tell the story about why there is stability and instability, which depends on the parameters within the

¹⁷ Abraham & Shaw, 1992, Figure 7.2.7. p. 243; also equivalent Figure 2.2.7, in Part II, 1984, p. 42.

equations of a system that make its various terms have a stronger or weaker influence on the system.

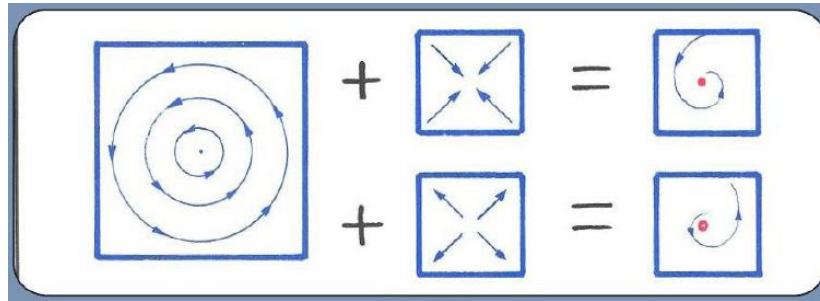


Figure 9. The *Nonhyperbolic, Unstable Center*. It is easily perturbed to a point attractor or repeller. It lies at the bifurcation point between them. It is neither attractor nor repeller until perturbed by a slight change in a control parameter. When the parameter is far from the bifurcation point the system is more stable; the closer it gets, the less stable it is until it becomes nonhyperbolic at the value of the control parameter that is the bifurcation point. This approach to the bifurcation point is often accompanied by an increased complexity (fractal dimension) and amplitude of the attractor.¹⁸

If the various factors are nearly equal, the system is easily unbalanced, but if they system has some terms with a much stronger influence, it is not easily unbalanced. Think of a teeter-totter with nearly equally sized individuals versus one with individuals of very unequal mass. **Exercise:** Make a portrait for one end of the teeter-totter, position versus velocity. **Hint,** see the portrait for the magnetic pendulum in Abraham & Shaw. Draw one portrait for nearly equal sized riders, and one for very unequal sized riders. In addition, maybe also draw one for exactly equal sized riders. Where does the teeter-totter stop?

The complex numbers of the Argand Diagrams (which perhaps should be called the Wallis- Bouée-Argand Diagrams) are usually expressed in the form $a + bi$; a, b real, $i^2 = -1$. With respect to these diagrams, the concepts of stability and instability are also very nicely summarized for two dimensions by Sprott:

“If all the eigenvalues lie off the imaginary axis (have $A \neq 0$), as they do for all the cases except the center, then the system is called hyperbolic. If any eigenvalues have zero real parts, then the system is nonhyperbolic. The eigenvalues move around in the complex plane as the system parameters change, and a bifurcation occurs whenever an eigenvalue crosses the $A = 0$ axis or moves onto or off the $B = 0$ axis.” (Sprott, 2003, p. 75)

10. CHAOS

Now we come to one of the most fascinating topics in dynamics, that of chaos theory. One of the most iconic dynamical systems is the Rössler system. Figure 10 shows it with typical parameters yielding a chaotic attractor. (Rössler, 1976.)

¹⁸ Abraham & Shaw, 1992, Figure 12.1.8, p. 368; also equivalent Figure 3.1.8 in Part III, 1985, p. 40.

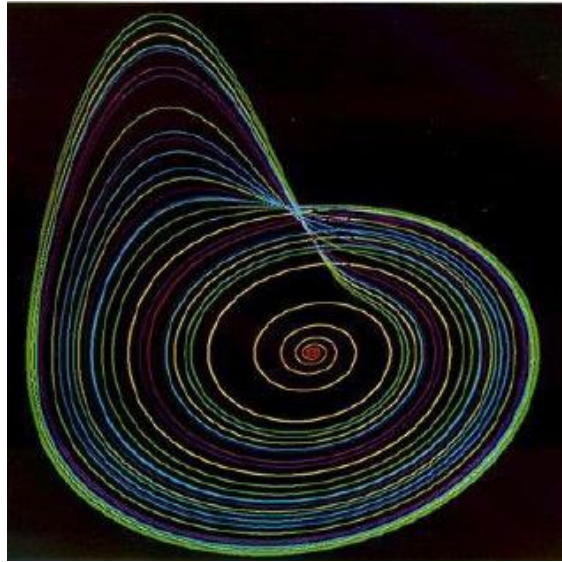


Figure 10a. Rössler Attractor¹⁹. Trajectories spiral outward on the x-y plane from a saddle point (red dot). Another saddle point influences the trajectories back to the band.



Figure 10b (above). The Famous Meg Tattoo²⁰. Based on the Briggs' Figure, and Meg's.

The Rössler attractor exists in a three-dimensional state space and is organized around two saddles. The outset (of index 2 on the x-y plane) of one saddle (red dot of Figure 10) consists of trajectories that diverge from each other as they spiral outward counterclockwise. The second saddle point (not shown), upper right away from the attractive thick band of the attractor, pulls the trajectories up and turns them back toward the band. Figure 11 shows that the region occupied by a set of trajectories has expanded after one trip around the attractor.

¹⁹ Briggs, 1992, p. 141. Also see beautiful 3D rotating velocity-coded trajectory by Ralph Abraham at <http://www.visual-chaos.org/chaos-movies/WebGL/Lines%20Lab/Rosler-5000/my-clc-05.html>

²⁰ <http://www.megspohn.com/?p=560> Meg/s blog about her tattoo.

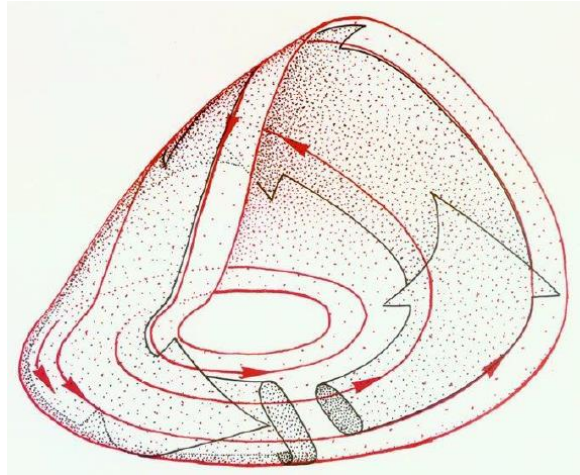


Figure 11. Expansion of a set of Trajectories in one turn around the attractor.²¹

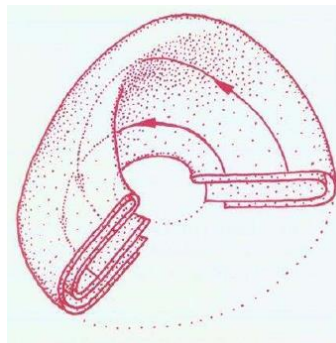


Figure 12. Fractal Microstructure.²²
Figure 12a (above). *Poincaré Sections*.
Perpendicular to the Rössler Band



Figure 12b. *Lorenz Section*
Perpendicular to the Poincaré Section.



Figure 12c. The *Lorenz Section* reveals the pattern of the layering of the Rössler Attractor.

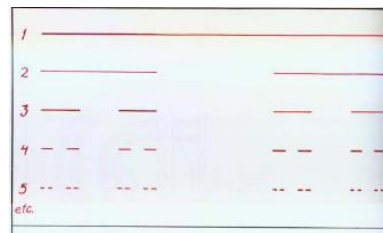


Figure 12d. The *Cantor Process*.
Successive steps of decimating a line
illustrate the self- similarity across scale
evident in the chaotic attractor.
Middle-thirds decimation shown.

²¹ Abraham & Shaw, 1984, cover.

²² Abraham & Shaw, 1992, pp. 319-320.

This process leads to an infinitely layered thick surface as a fractal pattern which is revealed when transected by planes, called *Poincaré* and *Lorenz Sections* (Figure 12).

The rate or strength of divergence of trajectories within and convergence to the attractor are given by *Lyapunov Exponents* (*characteristic exponents* and *multipliers*), one exponent for each dimension of a chaotic attractor, which comprise the *Lyapunov Spectrum*. Positive exponents measure the rate of divergence while negative Lyapunov exponents measure the rate of convergence. The absolute value of the negative exponents must exceed that of the positive exponents or the trajectory would depart the space with the result of no attractor.

Often the largest positive exponent is reported rather than the whole spectrum to give a quicker characterization the *sensitivity to initial conditions*, a defining feature of trajectories of chaotic attractors. The exponents may be computed locally or globally for an attractor.

The Rössler system started life as a set of three ordinary differential equations, but there are methods for *reconstructing attractors* and determining their *fractal dimensions* (the degree to which the state space is occupied), and their *Lyapunov Exponents* from experimental data using a handshaking combination of geometric reconstruction and numeric computation of the fractal dimension. (See Abraham, 1997 for a brief review; Abarbanel, 1996; Kantz & Schreiber, 1997; Ott *et al.*, 1994; and Sprott, 2003 for more complete details.)

The Equations for the Rössler System are:

$$dx/dt = -y - z; \quad dy/dt = x + ay; \quad dz/dt = b + z(x - c)$$

Rössler used $a = b = 0.2, c = 5.7$, but now $a = b = 0.1, c = 14$ are usually used.

Some Useful Links for the Rössler System

http://www.scholarpedia.org/article/Rossler_attractor

Christophe Letellier and Otto E. Rössler (2006), Scholarpedia, 1(10):1721. Shows and gives the location of the two saddle points that organize the attractor. Shows attractor in stereoscopic view and in rotation for better visualization, and shows the folding into layers. Gives the unimodal first-return map and the Lyapunov spectra. This is a compulsory site.

http://en.wikipedia.org/wiki/R%C3%B6ssler_attractor Presents properties via eigenvectors, Poincaré maps, and bifurcation diagrams.

<http://chaos.wlu.edu/106/programs/rosslerdes.html> Allows launch of an interactive applet which allows one to choose parameters and runs two trajectories on each 2D plane of the attractor.

<http://demonstrations.wolfram.com/TheRosslerAttractor/> The Wolfram demonstrations can be dramatic. They require download of the Mathematica Player. There is an interactive applet allowing changing the three parameters, which produces static graphs of time series for each dimension and the 3D attractor.

11. DYNAMICAL BIFURCATION THEORY

There are three basic types of bifurcations for the finite dimensional case: *catastrophic*, *subtle*, and *explosive*.

Catastrophic bifurcations, also called *blue sky bifurcations*, are those where a limit set (attractor) appears or disappears. The best known example is the cusp (Figure 5.) where a

second point attractor entered or left the one-dimensional portrait as the control parameters changed. Here is another example where a second periodic attractor appears or disappears.

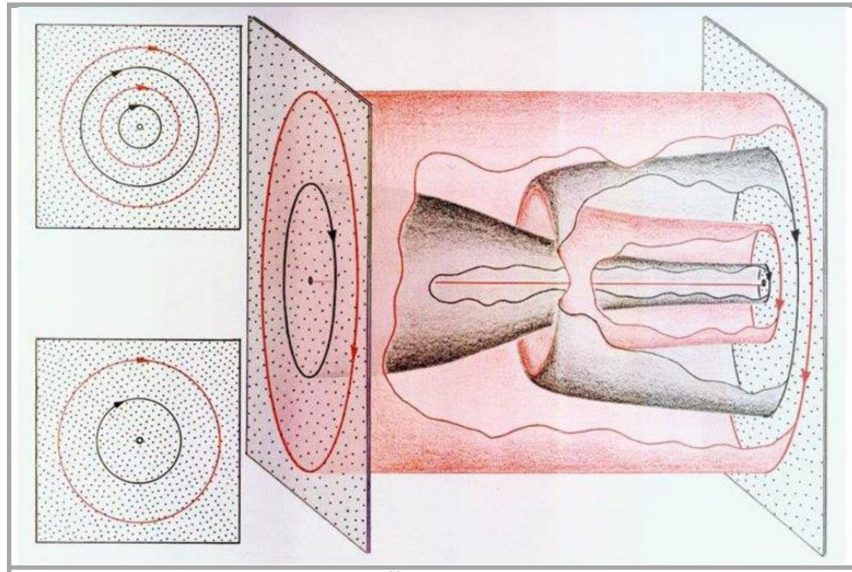


Figure 13. Periodic Fold Catastrophe in $2D^{23}$. *Attractors* (black), *separatrices* (red), *state spaces* (vertical planes), and the horizontal axis is the *control parameter*. As the control is moved from left to right, a second *periodic attractor* appears; as it is moved from right to left, it disappears. The inserts show the portraits at the ends of this sequence. The horizontal red line is the locus of the *point repellors* which are *virtual separatrices* in the center of the attractors; they do not separate basins.

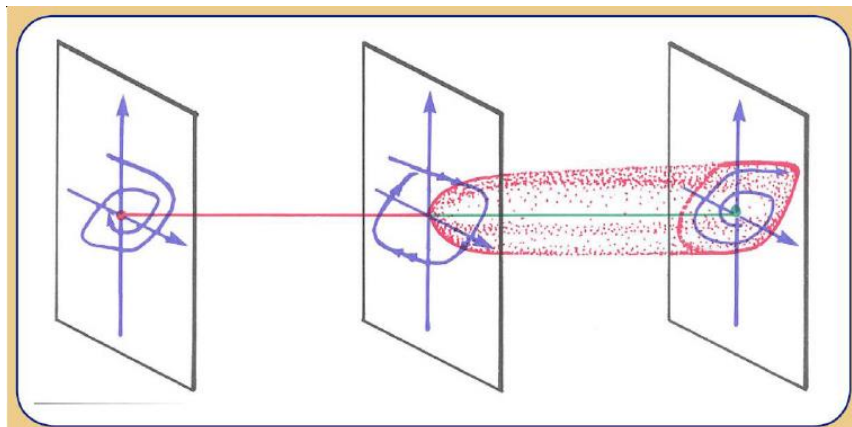


Figure 14. The Hopf Bifurcation²⁴. As the control parameter is changed from left to right, the spiral point attractor (left) becomes a center at the bifurcation point (middle) and then a periodic attractor with a repeller in its center (right) which continues to grow parabolically as the parameter continues to increase. This subtle bifurcation is also called the Poincaré-Andronov-Hopf Bifurcation, or Excitation.

²³ Abraham & Shaw, 1987, p. 566.)

²⁴ Abraham & Shaw, 1988, p. 172; 1992, p. 609; 2005, Figure 17.1.7, p. 495.

Subtle bifurcations are those where one attractor is transformed into one topologically (qualitatively) different but gradually in the sense of initially being in a small epsilon neighborhood of the previous attractor. Here is a prototypical subtle bifurcation. It was known to Poincaré in 1885, analyzed by Hopf in 1942 for the couette stirring machine, and has had numerous other applications such as the Van der Pol transmitter.

Explosive bifurcations involve a sudden large increase or decrease in the size of an attractor, usually involving the disappearance of saddles or a change in **heteroclinic connections** (saddle-to-saddle). They are varied and complex. Some are between point and cycle, some between cycle and braid, some between cycle and chaos. Here is one in which small chaotic attractors merge into a larger one, explosively and fractally smeared over time.

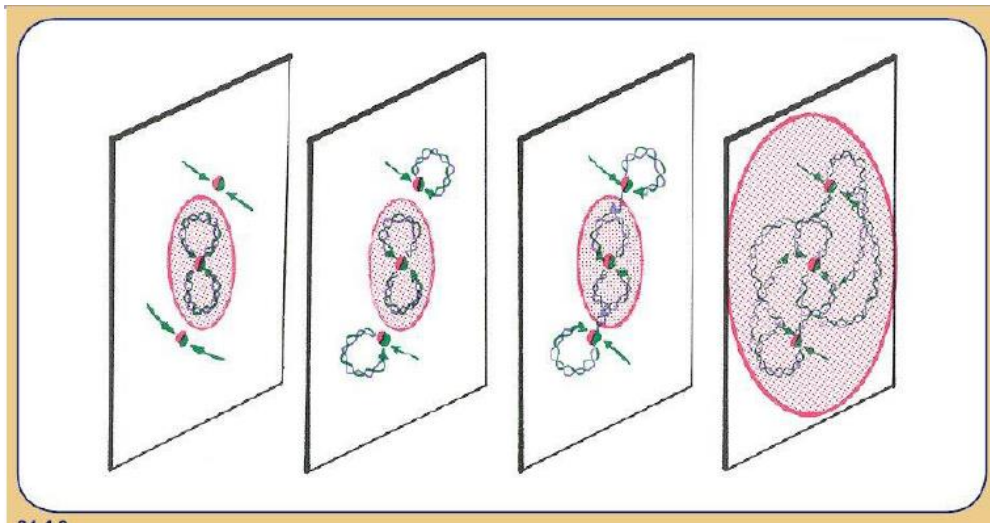


Figure 15. Ueda's Chaotic Explosion in 3D.²⁵ The attractor is in the red area. Outside of it are two saddles with *homoclinic connections* (from the saddle to itself). The chaotic attractor in the center is a *tangle* organized around a saddle. Trajectories from the outer saddles are attracted to the center saddle and, along with its own homoclinic connections, form the inset to the center saddle. Then the outset of the central saddle makes connections to the outer saddles, which are then part of the larger chaotic tangled attractor. It is an explosive chaos-to-chaos attractor.

12. SELF-ORGANIZATION, NETWORKS, AND CONTROL THEORY

These topics are of paramount importance, but we mention only the minimum of ideas leaving these topics for further exploration. Self-organization refers to the fact that one or more control parameters of a system are a function of the state of the system and thus the behavior of the system will change accordingly. The changes may be subtle or gradual, or if a bifurcation point is passed, they may be dramatic, i.e., non-topologically equivalent.

²⁵ Abraham & Shaw, 1988, p. 172; 1992, p. 609.

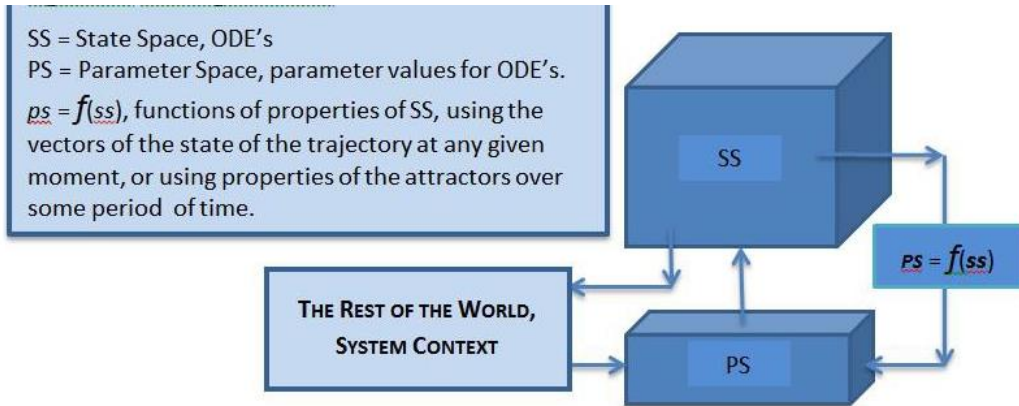
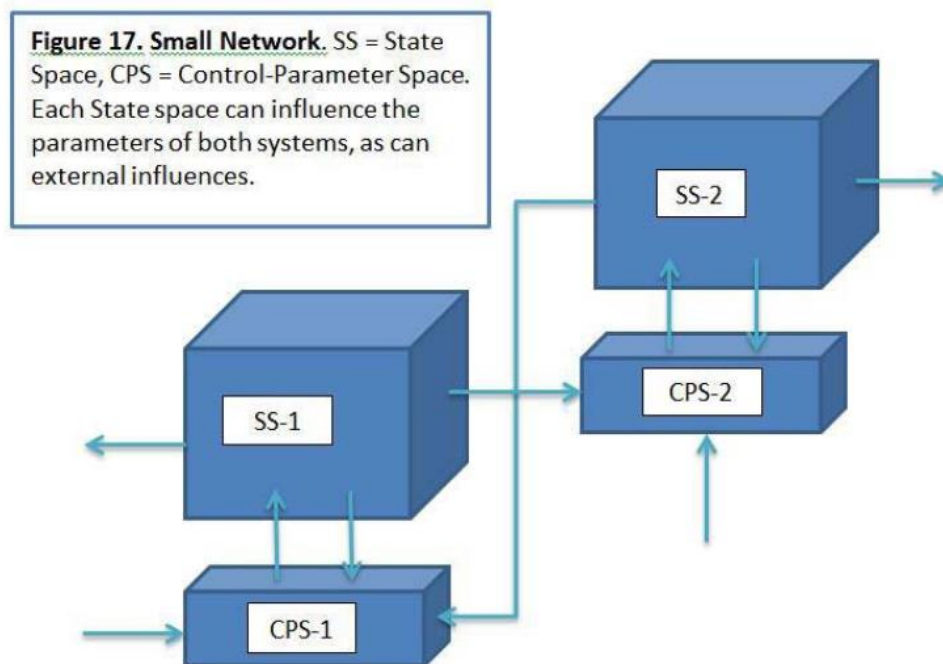


Figure 16. Self-Organization.

One can have multiple dynamical systems interconnected in various configurations, which we have previously called *complex dynamical systems* when the systems are fairly homogeneous in type. *Complexity Theory* may be considered as a superset of Systems Theory extending some of the basic ideas to hybrid networks that may include continuous and discrete systems of a greater variety than those of the nonlinear dynamical systems we have been discussing here. There are many varieties of these, but the most generalized and versatile of these include *complex adaptive systems*, *agent-based systems*, and *scale free networks*.

Here is a representation of a very simple network.



In our scheme, *self-organization* is simple conceptually, but allows for complex outcomes. Self-organization is where a system can create changes, including bifurcations, by

influencing its own control-parameters (we could call that *homoclinic self-organization*), or where its influence is mediated by its influence on other components in the network, with one or more of those feeding back onto the control-parameters of the first referenced component we might have designated (we might call that *heteroclinic self-organization*).

We believe that our definition of self-organization which is based on that of Abraham and Shaw (1982) fits Ashby's (1962) definition though it fits neither of his cases of self-organization, that of either a changing evolution of connectivity among subsystems within a network, or the evolution toward a 'good' organization of a system. That is, our definition allows exploration of parameter space to investigate the various behaviors a system is capable of without necessarily directing the system to a good or bad organization, leaving such decisions to *a posteriori* evaluations.

Intentional systems, in our view, include those that can navigate in parameter space and state space (as depicted in the basin-boundary portrait—aka response diagram), and thus induce bifurcations (Abraham, 1994a; Kugler *et al.*, 1990; 1991). That is, such systems can make choices among future attractors. This would seem to imply a self-awareness of at least some aspects of both its state space and its parameter space, and maybe some of the whole network, and perhaps of the network's context (in which case the aware agent is both a part of the system, and of its context). We very briefly discuss this paradox below.

Emergent properties of the network can flow from these self-organizational features. As Goldstein (1999) puts it, emergence is "the arising of novel and coherent structures, patterns and properties during the process of self-organization in complex systems." Of course with our definition, emergent simply means we, or nature, has not yet explored the whole of the parameter space before, so we see system behaviors being realized for the first time. Figure 6 could be an example; Chalfan pushed the limits of the parameter space, and produced this beautiful fractal (Figure 6) never before seen, at least to our knowledge. Rössler has also pushed the logistic equation to new places, i.e., expanded the visibility of the state space.

Control of Chaos refers to the modification of the trajectory of a chaotic system in real time. A small modification or series of modifications of the trajectory is made based on some parameter of the attractor or the state of the system. Since the equations are usually unknown for a real complex system, the parameters are measures on a reconstructed attractor (see 10. Chaos, *supra*). Using these parameters, one can design systems to insure the evolution of the system some toward some target subset of properties of its behavior, its attractors. Figure 18, a revised version of the self-organizational Figure 16, outlines the process. The system may or may not be considered intentional, but may, of course, reflect the intention of the designer, if there be one (Ashby's 'good organization', 1962).

Most of the theoretical developments have been concerned with taming unruly chaotic behavior to more stable chaotic behavior, to regimes of periodic or near periodic behavior, or to fixed point behavior. (Boccaletti *et al.*, 2000; Ott, 2006).

A chaotic attractor can be characterized not only by its portrait or time series, but by its frequency spectrum (Figure 19). These frequency spectra reveal often some dominant frequencies and a mess of lesser ones. While control theory has been around for a while, Ott, Grebogi, and Yorke (1990) launched the contemporary exploitation of features of chaos to attenuate, enhance, or optimize some constraints to chaos in complex systems, in short, to sculpt more desirable system behaviors from the attractor. They start with a closer look at those multiple frequencies (unstable periodic orbits, or UPO's) comprising the chaotic trajectories. They noted that UPO's were of varying instability. Furthermore they exploit the

fact the unstable orbits occur ergodically in some sort of stochastic sequence, something like states in symbolic dynamics. These properties allow the periodicities to be evaluated from Poincaré sections of attractor reconstruction. A point representing an UPO to be targeted for stabilization is a saddle point. Its inset is a stable manifold, a separatrix between two basins of attraction, which is repelling. Its outset is an unstable manifold, which is attractive. Their local properties can be measured with Lyapunov exponents (aka eigenvalues, characteristic exponents; see Sections 8 **Characteristic Properties of portraits**, and 9 **Stability**, supra).

Parameters based on these can be used to move the trajectory closer to the stable manifold. Pushing around these trajectories by small increments, by dint of the exponential divergence of nearby trajectories in chaotic attractors (sensitivity to initial, i.e., nearby, conditions), could move the trajectories to the desired positions and behaviors faster than traditional linear methods. These adjustments are made when the trajectory approaches near a point in the neighborhood of the UPO to be stabilized (clamped). In short, this OGY method captures the trajectory in a range of frequencies close to the target frequency once it has wandered close.

Chaos thus enables flexibility in switching among favorable targets. (Baccaletti et al., 2000; Ott, 2006.)

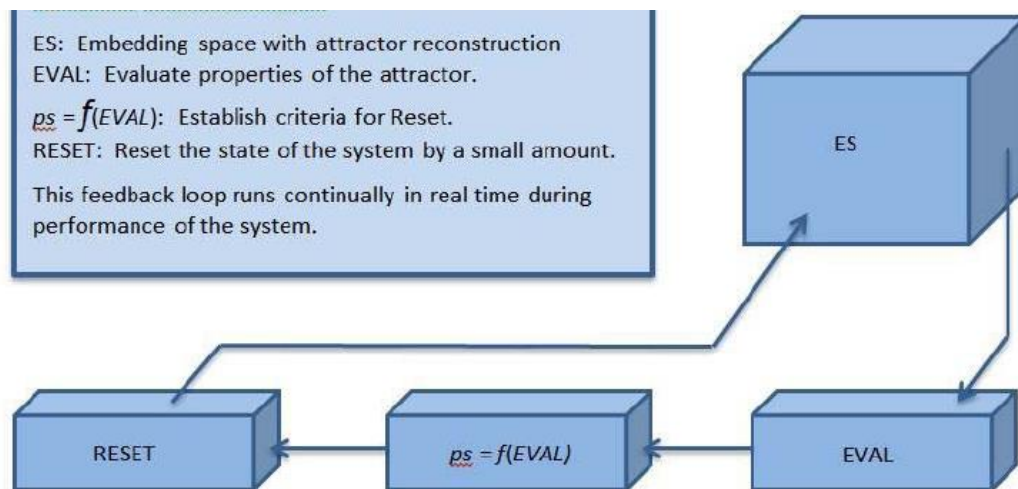


Figure 18. Control of Chaos.

There have been many instantiations of the OGY theory and modifications of it, including a magnetoelastic ribbon (Ditto et al., 1990) and quantum entanglement (Chaudhury et al., 2009). One of the most intriguing to us, not only because it is a biological system, isolated cardiac tissue (under chaos-inducing ouabain-epinephrine perfusion), but also because of the clarity of their procedure in showing visualization of their Poincaré sections (aka maps) and procedures (Garfinkel et al. 1992). They used electrical recording and stimulation, and then constructed the Poincaré maps as return maps of the interheartbeat intervals (I_n). This abbreviates the attractor reconstruction process to a single time-delay step

(see Section 10 Chaos, supra)²⁶. Their Figure 2 (within our Figure 20) well illustrates the three-step control process of EVAL—ps— RESET of Figure 18 supra.

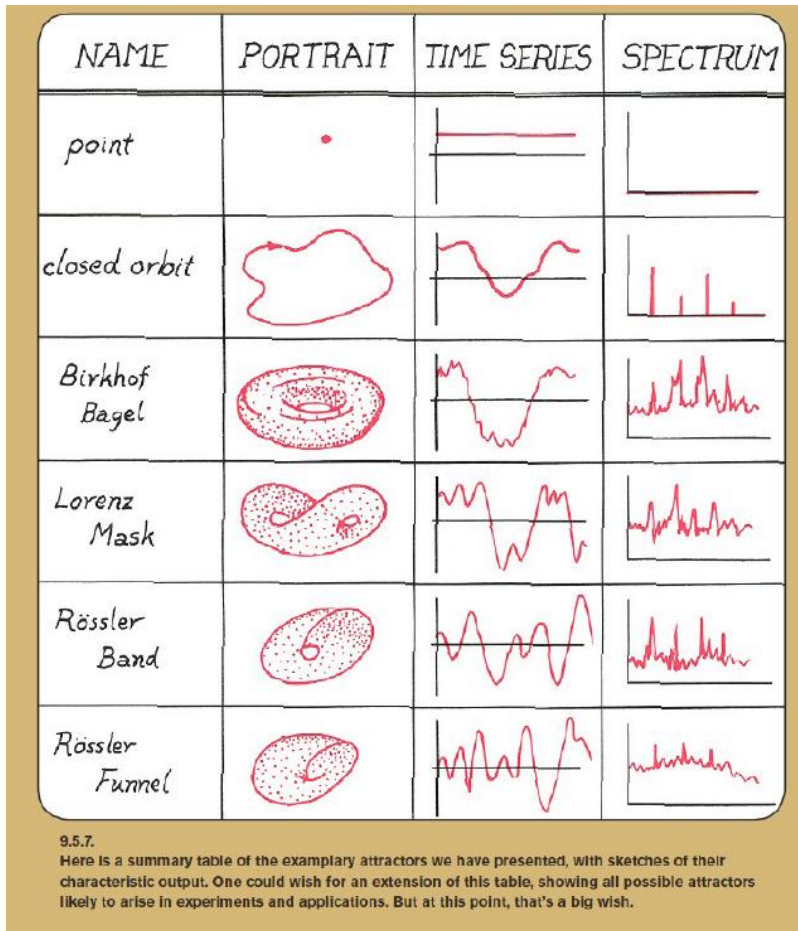


Figure 19. Portraits, time series, and frequency spectra of point, periodic, and chaotic attractors. (From Abraham & Shaw, 2005, Part II, p. 327.).

Another prominent method also involves evaluating periodicities for the trajectory of a given state variable for both measurement and direct control of the state of the system with a continuous time-delay feedback loop. When a copy of the signal time-delayed by a time, τ , equal to the period of the target frequency, the variable is adjusted to that of the delayed value. (Pyragas, 1992); Pyragas & Tamasevicius, 1993).

There are many other methods, and work that has extended the Poincaré analysis, which can be pursued starting with the citations here. We also wish to emphasize that while the control methods were developed to target periodic orbits, that these obviously generalize to

²⁶ The sequence previously mentioned, handshaking in attractor reconstruction, is comprised of alternating the addition of delayed time series with evaluation of the fractal dimension at each step until the fractal dimension and plumping up the attractor reach an asymptote. Thus the 2D return map is equivalent to a Poincaré section of a 3D reconstructed attractor, without having gone to the trouble of making the 3D reconstruction or taking the process to the point of having a full reconstruction which might have required even higher dimension. In most instances the information may be sufficient for the control of chaos anyway.

target chaotic attractors as well, which are beneficial to many biological, social, ecological, psychological systems, as well as physical and mechanical systems. Or as Sprott (2003, p. 146) put it: “Sometimes it is desirable to use feedback to keep the orbits unstable (‘anti-control’ of chaos.)” We might further suggest that optimizing *to* chaotic regimes of mid-dimensional complexity makes for more creative and evolutionary processes (Alford & Abraham, 2011; Krippner, Richards, & Abraham, 2014).

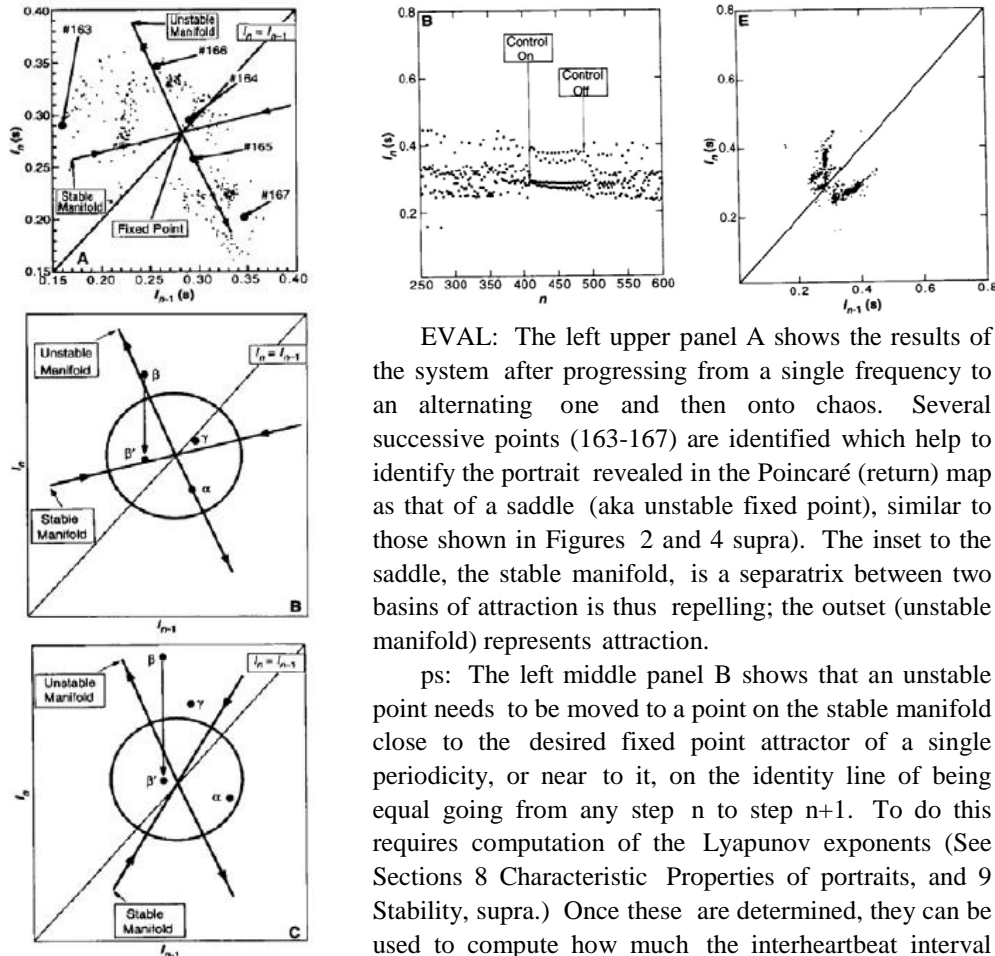


Fig. 2. (A) The Poincaré map of the aperiodic phase of a ouabain-epinephrine-induced arrhythmia in a typical heart preparation illustrating the structure of the chaotic attractor. (B) Blowup of the Poincaré map in the region of the unstable fixed point illustrating schematically the idealized chaos control method. (C) A schematic of the actual implementation achieved with chaos control. Note that the stable manifold has shifted, a common variation among data runs. The circles in (B) and (C) enclose the region about the fixed point in which chaos control is enabled.

EVAL: The left upper panel A shows the results of the system after progressing from a single frequency to an alternating one and then onto chaos. Several successive points (163-167) are identified which help to identify the portrait revealed in the Poincaré (return) map as that of a saddle (aka unstable fixed point), similar to those shown in Figures 2 and 4 supra). The inset to the saddle, the stable manifold, is a separatrix between two basins of attraction is thus repelling; the outset (unstable manifold) represents attraction.

ps: The left middle panel B shows that an unstable point needs to be moved to a point on the stable manifold close to the desired fixed point attractor of a single periodicity, or near to it, on the identity line of being equal going from any step n to step $n+1$. To do this requires computation of the Lyapunov exponents (See Sections 8 Characteristic Properties of portraits, and 9 Stability, supra.) Once these are determined, they can be used to compute how much the interheartbeat interval needs to be shortened, and a stimulus was given to precede the next beat to accomplish it.

RESET: The implementation is indicated in the left lower panel C. Above are results from one experiment: time series left panel B; Poincaré map right panel E, small dots for control off, large for control on. (From their Figure 3).

Figure 20. Controlling Cardiac Chaos (Figures 2 and part of 3 from Garfinkel *et al.*, 1992), by permission of Garfinkel).

13. PERSPECTIVES FOR PSYCHOTHERAPY

Here are just a few dynamical principles to evaluate regarding therapeutic practice.

A. Stability, Instability, and Bifurcation

One approach is to view the therapeutic process as one of trying to achieve a transformation of certain aspects of self²⁷. A person with a partly dysfunctional attractor within the self resists change despite maladaptive features due to certain advantages they seem to provide; collateral damage, side effects, so to speak. They can move to a bifurcation point by some combination of learning and emotional adjustment. The source of this point of view comes from the idea that the move toward the bifurcation point involves destabilizing some aspects (attractors) of self by using any identified control parameters, getting near to the bifurcation point, then bifurcating, and then stabilizing the new attractors, a process made famous by Kurt Lewin's 'unfreeze-change-freeze'. (1947). See Section 9 and Figure 9. Orsucci (2008, his Section 7), echoing Varela (1999), emphasizes that this is a continual process, a sequence of instabilities and bifurcations: "*constitutional instabilities are the norm and not a nuisance* to be avoided." This happens many times during the course of therapy, education, and a lifetime. Different authors and therapists approach this process metaphorically, some with formal models, and some with research.

My friend Derek Paar refers to this process as 'Noing' (saying no) and uses riddles to seed it (Paar, 1993). When Croesus, King of Lydia, consulted his therapists at Delphi and Amphiaraus about making war on Persia, they each replied ambiguously that to do so would destroy a great empire. Pythia (the oracle at Delphi) also told him that he should beware when a mule became lord of the Medes. Since it seemed impossible to Croesus for a mule to be a ruler, he interpreted this to mean that he would prevail over the Persians. This was a metaphorical trick taking advantage of Croesus' literal-semiotic mind. It did not occur to him that 'mule' could refer to Cyrus the Great, King of Persia. As a consequence, the great empire that was destroyed was his own, not Persia. (Herodotus, I.55). The purpose of this trickiness of Greek oracles has been interpreted as to reveal gods' ill will toward humans, (Manetti, 1993, chapter 2), but an alternative interpretation might be that these were lessons in getting humans to think for themselves, to be self-organizational, rather than to trust rigid, deterministic semiotic codes. I made a similar point in a mini-sermon about Eve that I gave at a ceremony for psychology students at Silliman University (Abraham, 1997a). Terry Marks-Tarlow has made a similar analysis of Pythia's two sessions with Oedipus before and after his solving the riddle of the Sphinx, a well-known story dramatized by Sophocles. She suggested

²⁷ No term adequately served my purpose here. I considered terms such as behavior, personality, being (Maslow, 1999), and self-system, but Stanley Krippner and Ruth Richards kindly pointed out their shortcomings for most of them. I conclude there is no adequate word to get to the idea that the self can be considered as a complex of attractors in flux, some more stable and durable, some changing at different times and with varying degrees of awareness. Probably the best perspective is that of neurological positivism introduced by Vandervert (1993), elaborated by me and apparent in the works of sociologists (Schwalbe, 1991) and Baumeister & Bushman (2011). "Neurological positivism regards phenomenology, brain, behavior, and environment as a holistic unity. Their complex interaction over time can be regarded as a complex dynamical system, which we might call mind. Attractors and other features of dynamical systems can represent processes of the mind." (Abraham, 1994b)

a self-referential transition to a more abstract level of thought (Marks- Tarlow, 2014), which is similar to my dynamical and Manetti's semiotic approach.

Such divinations finally became more secular, and led to divination evolving into discourse similar to that in the democratic polis. Another example also comes from Herodotus (VII.141) again involving Pythia. Some Athenian representatives consulted her concerning a Persian invasion, and were warned of disaster. The representatives refused the negative news and demanded a second more favorable opinion from her, refusing to leave until she gave them another answer, which she did. Her second and more complete reply led to open debate at the assembly in Athens over military strategy, showing that not only could one negotiate with the oracle, but that the oracle's pronouncement also led to public discourse (Manetti, 1993, chapter 2). Thus in the history of semiotics in ancient Greece and modern psychotherapy, there are many parallels which extol the place of the evolution of creativity and self-organization, as we shall see next.

B. Mid-Dimensional Chaos and Creativity

Most attractors are or can be considered as chaotic, especially those in the social and psychological domains. That implies that they exhibit both convergent and divergent characteristics (Section 10). My colleagues and I have suggested that creative process operate in a range of middle amounts of complexity reflecting a good balance of these convergent and divergent tendencies (Abraham, 1996, 2007, Krippner et al., 2014, Richards, 2007 and several authors therein; see also Guilford, 1953). In adequately defined attractors, these conditions are reflected in mid-range fractal dimensions.

This thesis has two implications for psychotherapy. The first is that the bifurcations within therapy will be best accomplished if these mid-dimensional conditions can be established. The second is that a major goal of therapy should be to establish a creative mode of life with flexibility, adaptability, some comfort with ambiguity, some comfort with challenge, tolerance, self-organization, and self-actualization, empathy, etc. In short, these goals are those emphasized in humanistic/existential psychology.

Since some of the changes are, at least in part, self-organizational, this means some dynamics of keeping within certain ranges of mid-dimensional complexity could employ some version of the control of chaos of Section 12; i.e., self-control.

Pauli had developed the exclusion principle before his collaboration with Jung and Jung had already been developing his acausal principle from a patient's dream about a scarab beetle. Despite the apparent complexity of quantum mechanics, I would maintain that its principles along with the lower dimensionality of the scarab dream and its related Egyptian mythology represent mid-dimensionality environment paving the way for their creative exploration of mind and matter. That is, many alternative explanations were excluded or discarded on the way, probably a process similar to that described in the control of chaos (in Section 12).²⁸

²⁸ See Burns, 2011, as one possibility for a start on this story. There is a wealth of other material available for those not familiar with it.

C. Evaluation of Initial Conditions

In chaotic attractors we have seen that 'sensitivity to initial conditions' predicates that in the near term, nearby trajectories tend to diverge. This fact can be relevant to therapy in two ways. The first is that evaluating the very first interactions between therapist and client should be performed carefully. I have another colleague, the Jungian, Michael Conforti (1999), for whom this is all import, and who includes the acceptance or rejection of/by the client as dependent on reading the nuances involved, and, if there is mutual acceptance, establishing conditions for the work to follow based on the initial conditions of the first interview with the client. The second is that it offers an alternative to changing system parameters to achieve bifurcations, to resetting the trajectory to new initial conditions. Instead of nudging aspects, parameters if you will, you find a different attractor-basin or different region within chaotic attractors, and boldly try to move the client to it and proceed from there. As an example of resetting initial conditions consider when Krishna tells his patient, Arjuna, to get a grip in order to get him to self-organize (*Bhagavad-Gita, II.38*).

Linda Dennard (2008) provides my favorite example of that I think exemplifies all these principles, and more, within an educational environment:

"In the first days of teaching at a university I would take an art print and tape it to the barren walls of the classroom — in particular a room that was tiered like a medical school arena — cold and clinical like an operating theatre. Over time, the paintings served to generate dialogue which could then be related to the course's topic, although that had not been the original intent. More, the picture broke the boredom and sterility of the classroom; remembering as I rolled out the scotch tape, that John Dewey deplored the artificial space of the American classroom. Indeed, Dewey made a connection between developing the capacity for self-governance and the existence of complex learning space filled with the provocative effects of nature and art. . . Dewey felt American schools were too much like bureaucratized, efficient prisons to be proper space for learning from the experience of democracy (Dewey, 2005, p. 21).

"To finish the story, each night after class, the janitor would take down the print; replacing it with a sign that said 'no posting.' Next class, the students would put up another print, but leave the janitor's sign in place. This went on until all the prints were gone and the walls were papered with 'no posting' signs. The students were amused with the determination of the maintenance staff to stay in control of the situation, as I realized that the janitor had been a helpful, if unwitting, participant in an object lesson on the adaptive dynamics of regulatory government. The lesson? Over time, the administration of democracy endures even at the cost of emerging elements of a social system that may actually transform the conditions that limit the evolution of democracy."

So, Linda and her students consulted the oracle asking if putting posters up would prevail, to which the oracle replied, "If you persist, the wall will be covered with posters." So they persisted only to discover that the posters that prevailed were those authorized by the campus bureaucracy, that is, in the wrong basin of attraction! And she surmised, as did I previously, that was a valuable educational experience. But did the students come away

Dewey-eyed? Let's hope so. I think the janitor, a trickster himself like Apollo, was having his fun as well.²⁹

D. Evaluation of Research

Has relevant research been done? Could it be done? Or would it be superfluous? As one tries to identify what therapeutic variables might or have been subjected to experimental definition, the trick will be to first estimate definitions permitting adequate time series. Then once having found at least one, to evaluate the number of dimensions involved. Many canonical multivariate analyses, variously called factor analysis, principal component analysis³⁰, eigenvalue decomposition, singular decomposition are available to evaluate the fractal dimension and its embedding dimension in nonlinear systems (see attractor reconstruction in Section 10 and Abraham, 1997). The idea is that since many variables are linked, the time series from one includes information of them all. So the next trick is to identify all those other principal variables after the analysis of the time delay embedding on a single variable has been run to saturation ('plumping up' the attractor and having the fractal dimension hit asymptote). Or doing well chosen multivariate research to begin with. Good luck!

Your research, or your search of the literature, or, of course, your experience, will have to guide you. There is much poetry and beauty to science and praxis than an algorithmic application of hard fast rules alone can supply.

14. THE ANTINOMY OF DYNAMICS

There was a debate over several years by Luhmann (1984), who pursued a systems' view of society and human affairs, and Habermas (1995), who evolved from Critical Theory to a pragmatic position of communicative competence in pursuit of a rational basis of emancipation. Both their arguments matured and came closer together during the course of their debate (Bausch, 2001).

The antinomy of dynamics is not inherent in dynamics itself, but forces itself upon us through our empathy with the universe, in whose bosom we are cradled, and from which we seek to escape. The semiotics of dynamics enables the metaphoric expansion from a closed system of equations to intercourse with the context of the system. This is the gateway to many arenas of philosophic discourse, which we avoid. However, one point is worth making now.

The apparent paradox is that most of us became very enthusiastic about dynamics, not only because it presented itself as a metamodelling strategy for science, but because it made us aware of our connectedness to a universe in a way that seemed immediate and understandable. It was almost a cosmology, and of course, it has been used to model cosmologies. On the other hand, any given model was a closed system, a finite set of n variables and their ordinary differential equations. Attractors can reflect the connectedness to

²⁹ I had a similar experience of having to teach in the most dingy room on campus (the faculty senate room, which explains a lot!!); so bad it was impossible to dispel the gloom of my students and myself, so we moved outdoors to an amphitheatre among a grove of beautiful acacia trees. Pythia was priestess to Apollo, sometimes supportive, sometimes tricking the human effort.

³⁰ Principal component analysis was invented by Karl Pearson (1901).

variables not declared in our models, but conjecturing about what those variables are might be difficult. We can even compute the dimensionality (embedding dimension) of the state space, and the fractal dimension of those attractors and we can reconstruct the attractors, but identifying the unknown variables may remain elusive, especially when the coupling constants to them are very weak. That is not the issue here, although it is one of long standing on the philosophy of the interplay between observation and theory in science. Alcmaeon proposed “the semiotic principal of conjecture: ‘The gods have immediate knowledge of invisible and mortal things, but men must conjecture (*techmaíresthai*)’ [that is human knowledge had to proceed] by evidence and conjecture.” (Manetti, p. 44).

What might be of interest is that, as Figure 16 depicts, from our conception of dynamical systems, the parameters and context of the system may interact. The system is context sensitive. Thus the paradox remains. The context become part of the system; a component of the network which attempts to evade formalization, similar to the failure of axiomatic number systems to behave without self-reference as Gödel’s (1992) incompleteness theorem showed. Also it is similar to the Russell’s-Zermelo paradox of the set of all sets that are not members of themselves and Russell’s theory of types (Russell, 1903), Quine’s response “. . . a repudiation of our conceptual heritage . . .” (1966, p. 11), and ‘type-free methods’ of dealing with paradoxes (von Nuemann, 1925; Quine, 1967; Atkins, 2013). There is also a similarity to Peirce’s semiotic triad, where context becomes a dynamic component (Irvine & Deutsch, 2014).

Here are some snapshots of that evolution of the hermeneutics of semiotics. Early Mesopotamian culture displays this sensitivity to context in the beginnings of formal representation of meaning into written languages. In cuneiform, the compound of the symbols for woman and mountain meant slave, because the context of women from the mountains implied slavery (Manetti, 1993, p. 3).

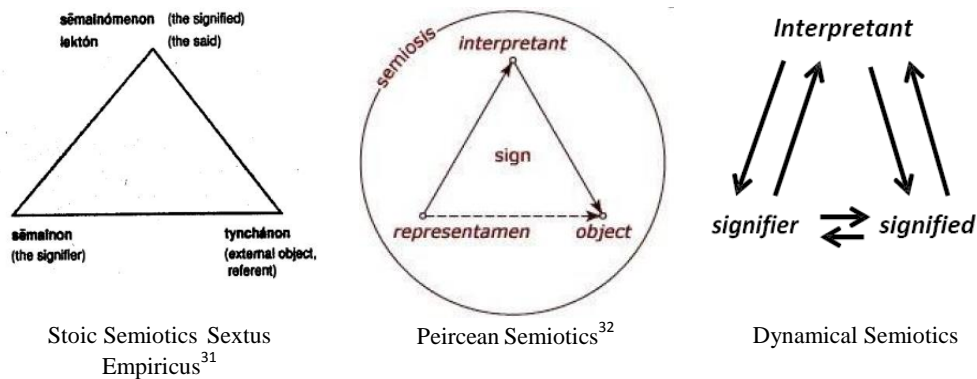


Figure 21. Semiotic Evolution.

The semiotic network could become a 3D state space, or remain represented as a network, with each node representing not just a single dynamical system at each node, but yet another complex network. The diagram for dynamical semiotics suggests that all components are interactive.

³¹ Manetti, 1993, p. 94. See also Sextus Empiricus, *Adversus Mathematicos*, VIII, 69-70.

³² <http://www9.georgetown.edu/faculty/irvinem/theory/peirce-semiosis.gif> Peirce, 1938.

If we consider semiotics as a meta-modelling strategy, that is, that language is used to represent our view of the world, and dynamics as a meta-modelling strategy, then it follows that we can represent semiotics dynamically (which Hardy, 1998, does; also Chandler, 2009; and Sandararajan & Kim, 2011). Moreover, in turn, semiotics can treat dynamics as a language. That is, semiotics can represent dynamics. Escher's (1948) hands that draw each other?

ACKNOWLEDGMENTS AND DEDICATION

We wish to acknowledge the intellectual and personal support from our diverse friends in liberation science: Linda Alford, Jay Allen, Dick Bird, Linnea Carlson-Sabelli, Jerry Chandler, Tina Champagne, Mike Cole, Allan Combs, Bill & Gail Corning, Linda Dennard, Eva de Lourdes Edwards, Neil Edwards, Bob & Connie Eldridge, Mark Filippi, Doris Fromberg, Martin & Pat Gardiner, Tom Gentry, David Gibson, Al Gilgen, Robert Gregson, Sally Goerner, Christine Hardy, Gus & Vic Koehler, Stanley Krippner, David Loye, Tom Malloy, Terry Marks-Tarlow, Frank Mosca, George Muhs, Judith Nagib, Rick Paar, Lothar Pickenhain, Bob Porter, Karl Pribram, Paul Rapp, Hector Sabelli, Ruth Richards, Robin Robertson, Henry & Carole Slucki, Meg Spohn, Clint Sprott, Carlos Torre, Karen VanderVen, Norman Weinberger, Enrico Wensing, and Tobi Zausner, many of whom are members of the Winter Chaos Conference and/or the Society for Chaos Theory in Psychology and the Life Sciences.

Special thanks to Ralph Abraham, Alan Garfinkel, Stanley Krippner, Ruth Richards, and Linda Alford for helpful comments and corrections, from copy edits to deep mathematical and philosophical insights. This article started life in response to a request for a chapter for a book, by Hector Sabelli, whose death rendered that project unfinished. I dedicate this article to Hector, for his passion for dynamics and philosophy, but mostly, for his passion for human rights and justice.

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