Commentaries on Albert-László Barabási's books

Networks101Link6.2   The 80/20 Rule Addendum on Power Laws
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The potential of the dynamics of power laws

Barabási emphasizes that the instability at bifurcation points in physical systems are characterized by power laws. I consulted Sprott's book so see how these applied to continuous physical systems, to find that he referred to Langdon's classic 1980 paper on cellular automata and the 'edge of chaos', so I was back in the wonderland of networks. Sprott did add some really interesting insights. Power spectra of chaotic attractors show a profile of noise obeying 1/f power spectra, with some peaks for the inclusion of quasi-periodic components (see figures in Abraham & Shaw; Abraham, Abraham, & Shaw). But Sprott points out (2003, p. 223) that "an abrupt change in the power spectrum as a control parameter is varied suggests a bifurcation in a system rather than noise." This is much in line with Barabási's contention that the dynamics at the bifurcation point involves a complex self-organizing dynamic.

Some of these complexities in networks is elucidated by Langton (1990) and several others (Packard, Mitchell, Crutchfield, Kauffman to name a few, mostly the SFI cabal) following up his work on the 'edge of chaos'. He worked with cellular automata, where there is a set of rules for how a the state of a discrete node depends on its neighbors, the same rule space being applied to every node in the network (usually a lattice or circle of nodes) as it synchronously goes through a sequence of iterations. His control parameter, λ, was the proportion of rules changing a node to a designated state. Thus if it were a binary network and a node was reading 5 neighbors including itself, then the neighborhood could be in 2^5 or 32 states; if the rule space said they all sent the node into state 0, then there would be a point attractor in one step, but if the proportion λ were .5 (16 rules to state 0, 16 to state 1), then eventually a chaotic attractor would form after a great many more steps. There was the usual bifurcation sequence of point, to cyclic, to chaotic attractor, except that in many cases a series of complex phenomena were observed before a rather random chaos appeared, and this intermediate stage (they elaborated on Wolfram's category IV, Li, Packard, &Langton, 1990). The complexity within those transition ranges were measured by spreading ratios, entropy, and mutual entropy, which plotted for several simulation show a fairly smooth increase from the periodic to random phases, but plotting individual runs show abrupt transitions occurring a slight different values of λ. With those periods, sever different patterns might appear in different parts of the state space, some chaotic, some empty, some periodic, changing at different times. These cellular models, of course, are much too simplified to model many interesting phenomena, but played a great role in the development of useful strategies in theory and research. Some of the implications are nicely made by Langton (1990, p. 35) are about evolution but apply to most self-organizational systems:

"Now nature is not so beneficent as to maintain conditions at or near a phase transition forever. Therefore, in order to survive, the early extended transient systems that were the precursors of life as we now know it had to gain control over their own dynamical state. They had to learn to maintain themselves on these extended transients in the face of fluctuating environmental parameters, and to steer a delicate course between too much order and too much chaos, the Scylla and Charybdis of dynamical systems. Such transient systems must have "discovered" how to make use of their intrinsic information processing capability in order to sense and respond to their local environment. Evolution has been the process by which such systems have managed to gain local control over more and more of the environmental variables affecting their ability to maintain themselves on extended transients with essentially open futures."[1]

He goes on to conclude, along with von Neumann that self-organizational systems operate within narrow limit of not too much or too little complexity, in the vicinity of the bifurcation point. I have called this a preference for midrange dimensionality, as a result of research on aesthetics.

I think that network theory, sophisticated despite its infancy (I know, 'infancy' is not true, Euler was a bit ago, so were Ulam, von Neumann, Erdős, Rényi, and Rapaport, but compared to where it is going?), has an advantage over dynamical systems, which lies in its ability to have localized variations in rule space, that is local dynamics, that can communicate globally but are generative locally, can be varied in time and space more easily than simple dynamical systems.

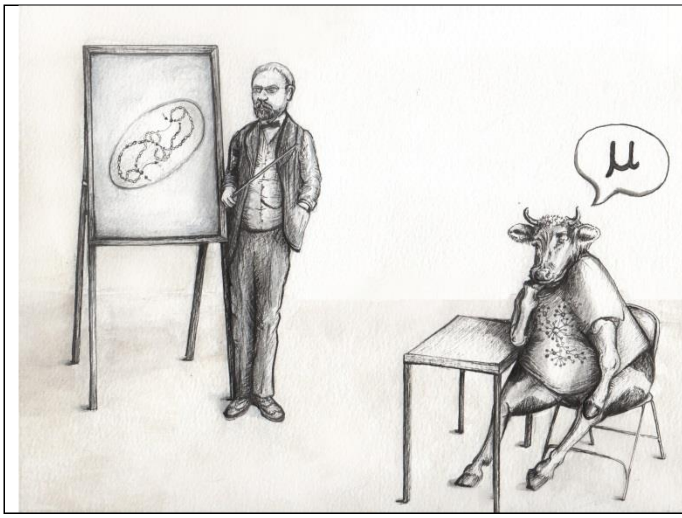
Further aspects of the relation of ordinary dynamical systems and networks

I've been working on an article showing parallel features of dynamics exhibited in their representation in network diagrams, time series, state spaces, and in their bifurcational properties. But Barabási and Sprott are suggesting a more intimate relationship. Dynamics point to the instability near the bifurcation point due to the control parameters providing more equal cooperative/competitive interaction among system variables. Barabasi's discussion of water molecules, magnets, quantal spins suggested that such dynamics were playing a role among those actors in their intricate dances. That is there were local networks with changing alliances among nodes. Both ordinary continuous dynamical aspects of communication within

[1] Dynamicists should be aware that network folks use order and chaos and randomly sometimes without recognition that dynamics treats chaos has having order, with randomness being at the limit of high dimensionality.

and between local nodes of agents contributed to global network dynamics, that is they were complex adaptive systems.

There are two other aspects of these dynamics. One is that in my essay, I campare the Ueda explosion to the Langtonian 'edge of chaos' as cascades of micro bifurcations into a macro bifurcation within a narrow range of control parameters, lumping them together in what I termed a μ-bifurcation (these are both represented in the next figure, with the Ueda on Poincaré's easel, and networks on the t-shirt; the Ueda explosion is represented in Abraham & Shaws work, and included in my article). I also suggest that most of the interesting bifurcations are chaos to chaos, and that the sequence point, cyclic, chaos is but one of many routes.



Musing over the μ-bifurcation

Poincaré's easel displays the Ueda Explosion as depicted in Abraham & Shaw. The t-shirt displays a preferential attachment network. The cow is one step ahead coming up with the μ-bifurcation idea and bored with the teacher's slowness. The artist is my grand niece, Laura Abrams. Many thanks to her and her gift.

Gregson (1988, 1995) has elaborated cascades of events involved in nonlinear perceptual dynamics based on his Γ recursion function, which includes a discussion of bifurcation in and out of chaos. (1995, p. 50.)

While chasing down some of this information relating dynamical systems to networks, I ran into an exciting example of their entanglement, that of the use of networks in getting information out of recurrence plots, which seemed opaque previously, and barely able to hint of bifurcation events (Donner *et al.*, 2010) . They take points from continuous time series in recurrence plots and treat those points as a network. Best we leave that for another time, as taking us a bit ahead of ourselves for now.

Limitations of Power-Laws and Scale-Free Networks

Invoking the power law as an explanatory or causal concept seems to me to be the fallacy of asserting the consequent. Power laws don't create dynamics, they reflect them; they reflect when dynamics they defies independence and randomness, when interconnectedness between actors, create outliers (fat tails) compared to the random normal distributions of independence.

Mitchell summarizes the potential of power laws and scale free assumption beautifully:

"So far I have implied that scale-free networks are ubiquitous in nature due to the adaptive properties of robustness and fast communication associated with power-law degree distributions, and that the mechanism by which they form is growth by preferential attachment. These notions have given scientists new ways of thinking about many different scientific problems" (2009, p. 253).

But then she goes on to say that "…skepticism is not only healthy, it is also essential for the progress of science." (*ibid*.) She then enumerates three critiques:

1. " Too many phenomena are [erroneously] being described as power-law or scale free." Barabási's work is given as an example! (*ibid*., p. 254.)
2. "Even for networks that are actually scale-free, there are many possible causes for power-law degree distributions in networks; preferential attachment is not necessarily the one that actually occurs in nature." (*ibid*., p. 254; see, I told you; fallacy of asserting the consequent.)
3. "The claimed significance of network science relies on models that are overly simplified and based on unrealistic assumptions." (*ibid*., p. 254.)

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