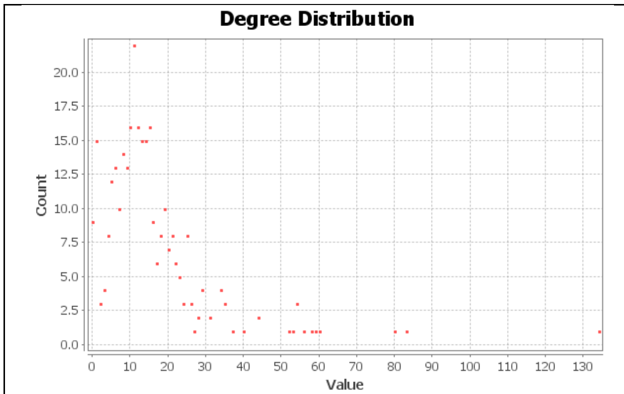
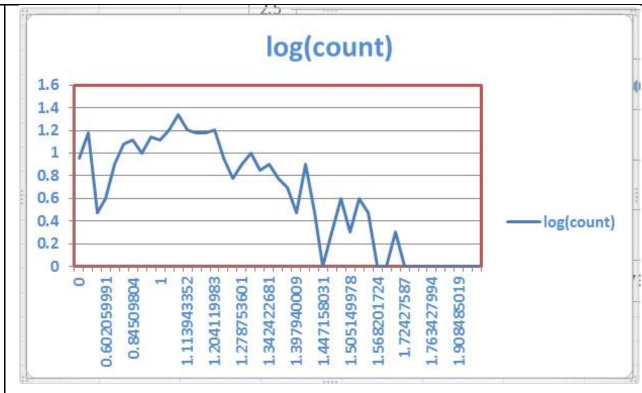
Commentaries on Albert-László Barabási's books

Networks101Link6.1   The 80/20 Rule ©frederick david abraham, 1 March 2013

Barabási makes the point that the distribution of degree web pages and many social networks follows a power law[i] rather than the normal distribution of Erdős-Rényi networks, the tails become more important. So I thought I would check that out for one of my pet networks, *C. elegans*. Here is my result.



Degree Distribution via Gephi

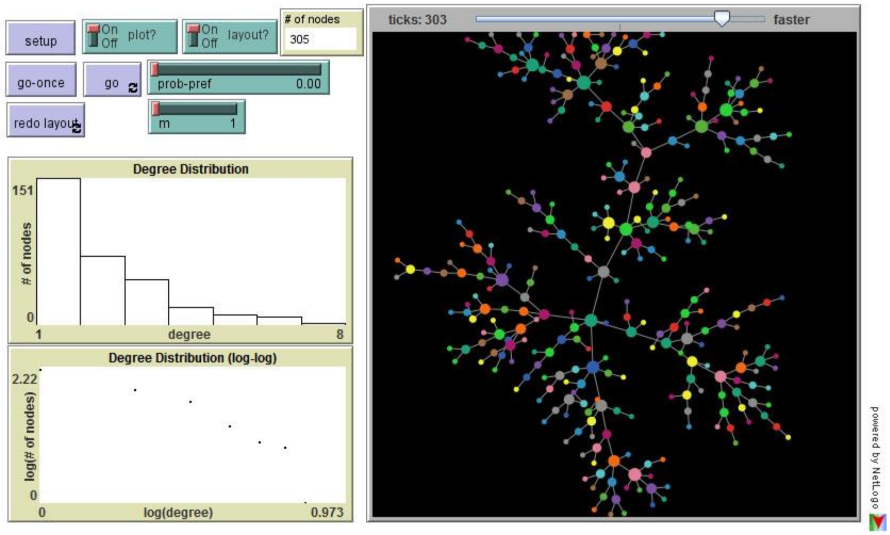


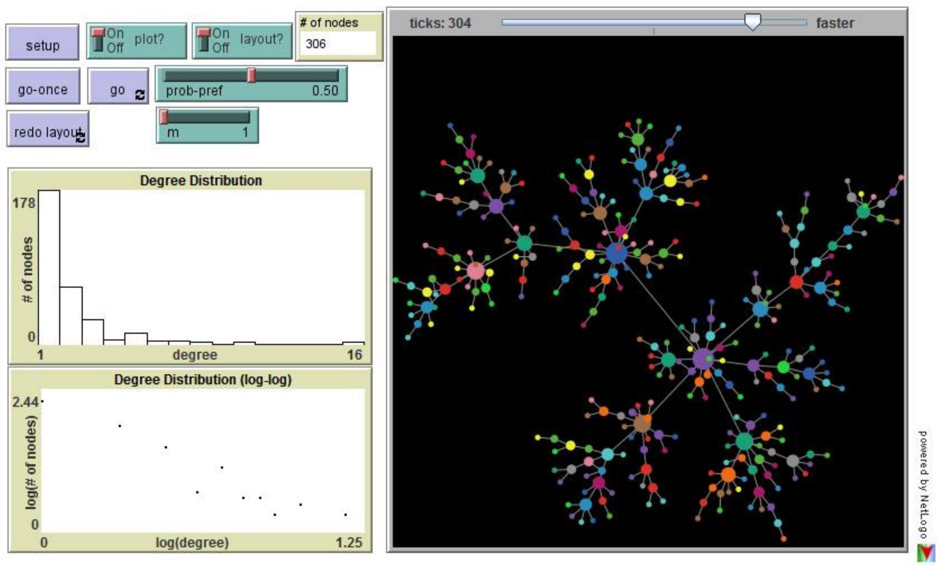
log-log plot via Excel

I would judge this to be intermediate between random (normal distribution) and scale-free (power-law distribution); it looks rather linear for degree >1 on the log-log plot. That is, it seems to hold when 10 or more nodes are involved, but nodes making fewer links may be following a different dynamic.

How does this fare with the 80/20 rule? The top 20% of the nodes can account for about 44% of the links. About half way to the 80/20 rule. There is less evolutionary preferential attachment getting to *C. elegans* than more scale free networks. Can anyone confirm or correct my estimates?

Next I explored the preference-probability parameter in Lara Adamic's[ii] adaptation of the NetLogo preference attachment model to see how the power-law slope for degree distribution and the graphic features of the networks varied.


setup
On Off plot?
On Off layout?
# of nodes
305
go-once
go
prob-pref 0.00
redo layout
m 1
ticks: 303
faster
Degree Distribution
151
# of nodes
0
1
degree
8
Degree Distribution (log-log)
2.22
log(# of nodes)
0
0
log(degree)
0.973
powered by NetLogo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On Off layout?
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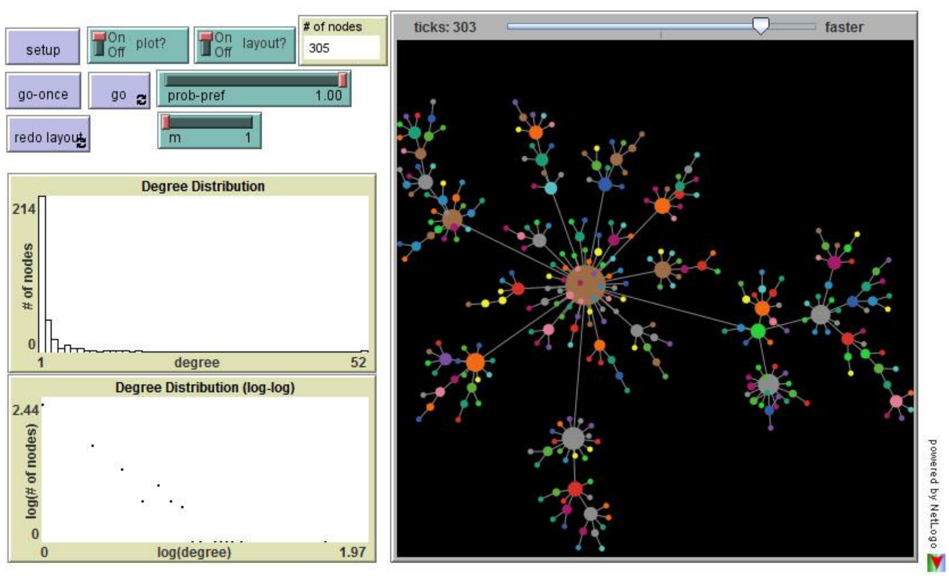
go-once
go
prob-pref 0.50
redo layout
m 1
ticks: 304
faster
Degree Distribution
178
# of nodes
0
1
degree
16
Degree Distribution (log-log)
2.44
log(# of nodes)
0
0
log(degree)
1.25
powered by NetLogo


setup
On Off plot?
On Off layout?
# of nodes
305
go-once
go
prob-pref 1.00
redo layout
m 1
ticks: 303
faster
Degree Distribution
214
# of nodes
0
1
degree
52
Degree Distribution (log-log)
2.44
log(# of nodes)
0
0
log(degree)
1.97
powered by NetLogo

Each simulation of the growth model was run to about the same number of nodes as elegans (~300). Note the increase in hubbiness as the preference parameter is increased from top to bottom, reflected in the increased kurtosis of the degree distribution, and the straight slope of the bode plot. Two things are clear about the preference parameter. Pr = 0 does not produce an Erdős-Rényi random graph, and, secondly, as the parameter increased, there is an increased affinity to attach new nodes to older ones on the basis of the older ones' size, i.e., their popularity or functionality within the nervous system, in short, their in-degree[iii].

End Notes

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[i] Also known as the 'Pareto distribution'. The term 'scale-free network' designates networks displaying this distribution.

[ii] http://ladamic.com/netlearn/NetLogo501/RAndPrefAttachment.html

[iii] The probability that a new node attaches to an existing node is proportional to the in-degree of the previous page. (Adopted from Easley and Kleinberg, p 548.) They summarize the dynamics of the random model and its normal curve and that of the self-organizational dynamics of the power law network thusly:
The normal distribution is generated through the independence of events via the Central Limit Theorem. But "Just as normal distributions arise from many independent random decisions averaging out, we will find that power laws arise from the feedback introduced by correlated decisions across a" [network]. (P. 547.)