Commentaries on Albert-László Barabási's books

Networks101Link4.1 Small Worlds ©frederick david abraham, 22 January 2013

There is not much to add here. Clustering is one of the things that differentiate random and biased networks, which thus adds to the usefulness of the comparison of network properties to those of random ones. Strongly biased links (strong ties) tend to create relatively smaller clusters, especially in using *triadic closure* a dynamics property of network evolution[i], but weak links (ties) can bridge the local to global properties of a network, create access to information that nodes in the smaller cluster do not possess—Granovetter's research. Formation a triadic closure was Watts' principal doctoral research question. To all this, I have little to add except to show my previous table, adding the clustering coefficients and numbers of triads and a comparison of the statistics for both directed and undirected interpretations of my exemplar networks, showing that it is easy to start playing with these things with real data, and thus learn more.

Network	$\bar{k}$	$\bar{d}$	**D**	**nodes**	**edges**	**C**	**TT**
c elegans (d)	7.663	3.992	14	306	2345	0.164	
c. elegans (u)	7.663	2.455	5	306	2345	0.308	3241
val (d)	11.619	2.523	8	307	3567	0.286	
val (u)	11.619	3.487	9	307	3567	0.595	21659
fred (d)	1.438	1.497	9	80	115	0.170	
fred (u)	1.438	2.907	6	80	115	0.679	117
lesmiserables (d)	3.299	2.400	5	77	254	0.287	
lesmiserables (u)	6.597	2.641	5	77	254	0.736	467
WWW Jan2013		20.35		1.35E+10			

$\bar{k}$	average Degree
$\bar{d}$	average path length
D	diameter
C	clustering coefficient
TT	total triangles
d	directed network interpretation
u	undirected network interpretation

A graph is considered small-world, if its average clustering coefficient $\bar{C}$ is significantly higher than a random graph constructed on the same vertex set, and if the graph has approximately the same mean-shortest path length as its corresponding random graph. [ii]

It is an interesting exercise to consider the variations in the average shortest path length, the diameter, the clustering coefficient, and total triangle between the different graphs and between the directed and undirected interpretations. The highest clustering coefficients are the (undirected) novel and the two undirected Facebook networks (Val, Fred). The drop in diameter us most pronounced in *C. elegans* which even with some electric synapses is very highly directed, so allowing the directed links to lose their directionality opens up many more new shortest pathways. The anomalous result for Val's Facebook diameter reflects the highly congested nature of the of her very interactive and social friends, seen in her huge number of edges and triangles.

End Note

[i] Easely & Kleinberg, (2009) *Networks, Crowds, and Markets*. 3.1 Triadic Closure, p. 48 and The Clustering Coefficient, p. 49. *Triadic closure: "If two people in a social network have a friend in common, then there is an increased likelihood that they will become friends themselves at some point in the future."* (P. 48.) They reference: Anatole Rapoport. Spread of information through a population with socio-structural bias I: Assumption of transitivity. *Bulletin of Mathematical Biophysics*, 15(4):523{533, December 1953.

[ii] Wikipedi, Clustering Coefficient. Retrieved 22 January 2013.