Commentaries on Albert-László Barabási's books

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Introduction to Theories of Networks.

Here we start to consider theories of networks that are not complex adaptive systems (CAS), which are networks of nodes comprised of dynamical systems (that is, each node is a set of simultaneous ordinary differential equations, or difference equations, or the equivalent) that deal with continuous variables, but rather deal with networks whose nodes are discrete variables of a few categories, most typically binary.[i]

One variety of network theory concerns networks that are made up of a fixed number of nodes and how each node changes, usually according to logical rules governing their responses to some fixed subset of those nodes. That is things exist in pretty much a fixed regular structure or lattice, and the only thing that changes over time are the states of the nodes. Much of this lineage is typified in the work on **cellular automata** (CA) and **Random Boolean Networks** (RBNs), "RBNs are similar to cellular automata, but with two major differences, nodes are [initially] connected not to spatially neighboring nodes but at random, and rather than all nodes having an identical rule, each node has its own rule."[ii] These models set the links and their rules, set an initial condition, and then set them into iterating. These networks have similar properties to dynamical systems, basins of attraction, bifurcation, and so forth.[iii] We shall not be concerned with these types of networks here, but they are tremendously important.

Erdős and Rényi[iv] (1959-1968) were concerned with a different process, that of how the networks are formed. This focus was not as concerned with what was communicated between nodes, or the rules of how nodes changed, but rather how were links laid down in the first place. Rather than seek solutions in deterministic rules, they suggested starting with a set of nodes and throw links between pairs of them chosen randomly, and see how the network depends on the number of link so strewn. These networks need not be constrained by a fixed number of nodes, but can add or subtract nodes as well. While this ". . . **theory of random networks**. This elegant theory so profoundly determined our thinking about networks that we are still struggling to break from its hold."[v] One of their most important findings was that there was a bifurcation from small isolated clusters of networks to a giant component involving a large proportion of the nodes as the average number of links per node increases to one, a finding previously described by Solomonoff and Rapaport (1951)[vi]. Another important finding is "That if the network is large, despite the link's completely random placement, almost all nodes will have almost the same number of links."[vii]The frequency of links per nodes follows a Poisson distribution (Bollobás, 1982).

The main concern is with how network structure evolves, whether by the random addition of links, or if by more deterministic rules, thus Barabási's skepticism as to whether that process is used by nature and societies. The nervous system of *Caenorhabditis elegans*, a parasitic nematode, has 306 nodes (neurons), 2345 links (via chemical and electrical synaptic paths), and an average degree of 7.663; our daughter's Facebook network is of similar size (307 nodes, 3567 links, average degree of 11.619), my own FB network being more pauce (but more comprehensible at 80 nodes, 115 links, and average degree of 1.438, due to unpopularity or being FB phobic; your choice). Obviously such nodes choose to dance together on some firmer basis than completely random (although some aspects of stochastic decision-making can influence the dance card).

While Erdős and Rényi (1959) recognized the limitations of their models for modeling real networks, the two major contributions as I see them are (1) Providing the mathematical modeling methods that are the foundations of the work that has followed, and (2) A random model to provide a null model to evaluate the amount of deviation from randomness that a network (real world or alternative models) might possess.

	
Paul Erdős	**Alfréd Rényi**

End Notes

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[i] Some of the early pioneers in this field include Conway (Game of Life in Gardner, 1970), Kauffman (1995), Langdon (1990), Malloy *et al*. (2010), Mitchell, Crutchfield, & Hraber (1994), Li, Packard, & Langton (1990), Ulam & Von Neumann (1940s), Von Neumann (1951, 1966), Weiner & Rosenblith (1946), Wolfram (2002).

[ii] Mitchell, M. (2009), p. 284. For the development of RBN's see Mitchell, *ibid*, pp. 281-287) and Kauffman (1995), both of whom discuss their significance for the study of the origins of life and genetic evolution.

[iii] Some have made such comparisons including Mitchell and Crutchfield, see unfinished ms by Abraham at http://www.blueberry-brain.org/dynamics/A%20Beginners%20guide%20II.pdf

[iv] Rényi was an activist under extremely dangerous circumstances, having escaped from a fascist labor camp, and later forged papers and wore a soldiers uniform of the fascist Hungarian Arrow Cross Party to help others escape.

[v] Barabási (2002), p. 14.

[vi] Solomonoff, R., & Rapaport, A. (1951). Connectivity in Random Nets. *Bulletin of Mathematical Biophysics, 13*, 107-117.

[vii] Barabási (2002), p. 22.