Commentaries on Albert-László Barabási's books

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The Königsberg Bridges' Problem

I was reading Euler's classic 1735 paper on the Königsberg Bridges' problem in which he credits Leibniz with first discussing topology ('geometry of position' or *geometria situs*). Euler stated,

> "In this paper I shall give an account of the method that I discovered for solving this type of problem, which may serve as an example of the geometry of position". (Euler, 1741)

[Euler's work was presented to the St. Petersburg Academy on August 26, 1735, and published as *Solutio problematis ad geometriam situs pertinentis* (The solution of a problem relating to the geometry of position) in the journal *Commentarii academiae scientiarum Petropolitanae* in 1741. I used the English version from Newman, *The World of Mathematics, Vol. 1*, p. 573.]

As most of you are aware, this was the real kick-off for both topology and graph theory. Also familiar is that he solved the problem by reducing it to a graph, using upper case letters (ABCD) to represent land masses, and lower case letters (a-g) as connections (now called ''vertices and 'edges' in graph theory, 'nodes' and 'links' in network theory). Also familiar is that his solution reduced the problem to that of a directed graph, and that the solution of such a problem would have to have zero or two, but no more, nodes of odd degree, and that the Königsberg bridges violated that theorem. His reasoning involved counting the number of visitations to each node depending on whether the node was odd or even. He simplified from there, stating these rules thus:

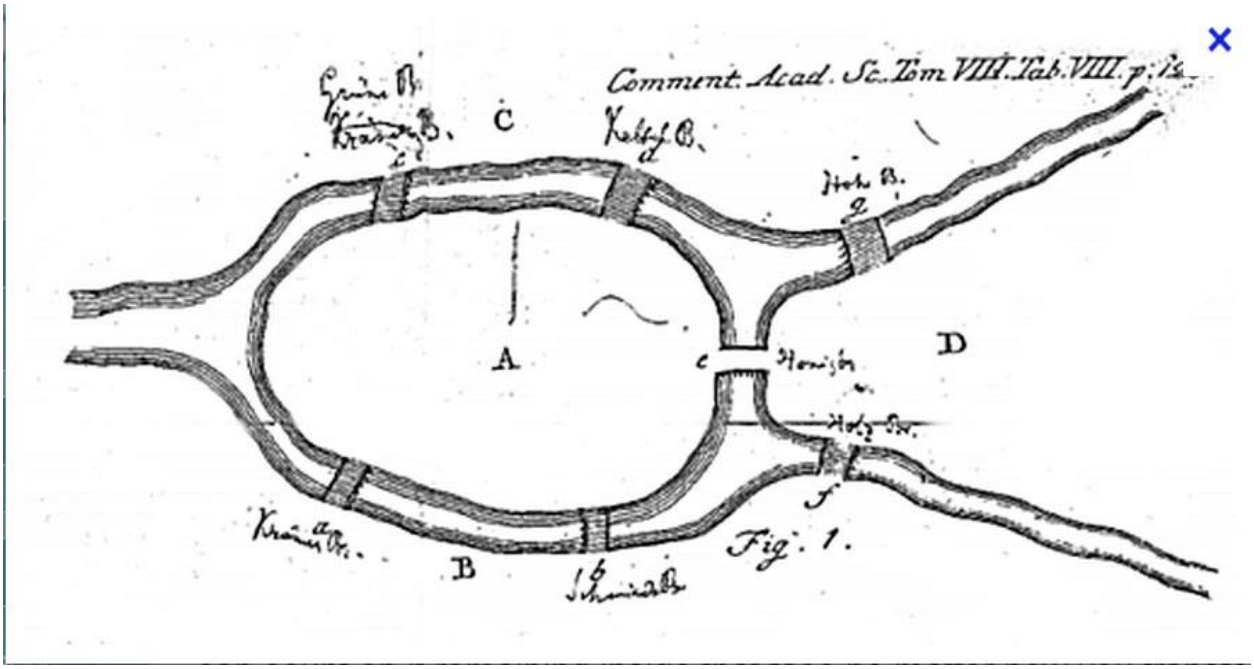
> "20. Thus for any configuration that may arise the easiest way of determining whether a single crossing of all the bridges is possible is to apply the following rules:
> "If there are more than two regions which are approached by an odd number of bridges, no route satisfying the required conditions can be found.
> "If, however, there are only two regions with an odd number of approach bridges the required journey can be found.
> "If, however, there are only two regions with an odd number of approach bridges the required journey can be completed provided one starts in one of the regions.
> "If, finally, there is no region with an odd number of approach bridges, the required journey can be affected, no matter where it begins. These rules solve completely the problem initially proposed." (In Newman op. cit., pp. 579-580.)

His article is very clear exposition and reasoning and reveals how he thought the problem out, but a contemporary statement of why this solution works might go something like this:

The bridges of Königsberg in Euler's day comprised a network of 4 nodes and 7 links; all nodes of odd degree, 3 nodes of degree 3 (3 land masses with 3 bridges) and one node of degree 5.

> "A person attempting to cross all three bridges [of one of the nodes of degree 3] only once has to visit [that] node at least twice. . .The problem is that he cannot leave again since there is no unvisited bridge left. Thus [that] node is either an end point or starting point of the journey." Thus if there are any nodes of odd degree remaining one cannot get there without traversing an already traversed bridge. (Précis and quote from Barabási (*Linked*, 2002), p. 244, endnote to p. 12).

Problem: In 1875 a new bridge was added. Where might you have put it, and as you start to check out whether it might work to yield Eulerian paths for tourists to enjoy, how could they pick a node at which to start, and determine where it would end? And how many Eulerian paths might there be?



Euler's Original Map for the Königsberg Bridges' Problem

[The Königsberg Bridge Problem was to find a path that crossed each and every bridge exactly once, an exercise in frustration, as Euler proved that it was impossible.]

Map from: http://www.google.com/imgres?imgurl=https://www.e-education.psu.edu/geog160/files/geog160/Excerpt%2520from%2520Euler,%2520L..png&imgrefurl=https://www.e-education.psu.edu/geog160/node/1949&h=256&w=500&sz=61&tbnid=na8ZSvlzDllyzM:&tbnh=72&tbn

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Leonhard Euler

Portrait by Jakob Emanuel Handmann (1718-1781
from: http://en.wikipedia.org/wiki/File:Leonhard_Euler_2.jpg